25 Chapter: Finite Simple Groups

The computer exercises in this section are based on material written by Christine Stevens at Saint Louis University. In this chapter we will use GAP to help us prove that A_5 and A_6 are simple groups.

The command `ConjugacyClasses(G)` lists all the conjugacy classes of a group G. For example:

```
gap> a4:=AlternatingGroup(4);
Alt( [ 1 .. 4 ]
gap> ConjugacyClasses(a4);
[ ()^G, (1,2)(3,4)^G, (1,2,3)^G, (1,2,4)^G ]
```

For $a \in G$, the notation $a\hat{}G$ above means the set of all conjugates of a in G. Thus we see that A_4 has four conjugacy classes: the conjugates of the identity, the conjugates of $(1,2)(3,4)$, the conjugates of $(1,2,3)$ and the conjugates of $(1,2,4)$. The command `ConjugacyClass(G,a)` creates the conjugacy class of G containing a:

```
gap> c:= ConjugacyClass(a4,(1,2,3));
(1,2,3)^G
gap> Elements(c);
[ (2,4,3), (1,2,3), (1,3,4), (1,4,2) ]
```

Exercises

25.1 Suppose you have disjoint sets T, U, V, W, X, Y and Z with cardinalities 1, 40, 40, 45, 72, 72 and 90 respectively. Suppose H is a set that is formed by taking the union of T with one or more of the other sets. List all the possible cardinalities of H. Which of these answers divide 360?

25.2 Use GAP to find all the conjugacy classes of A_6 and their cardinalities.

25.3 Let G be a group and H a normal subgroup of G. Let $h \in H$. Show the conjugacy class of h is a subset of H.

25.4 Use Exercises 25.1 - 25.3 to prove that A_6 is simple.

25.5 Use similar techniques as above to show A_5 is simple.

25.6 Show A_4 is not simple.

