1 Chapter: Introduction to Groups

The symmetries of a regular n-gon is called the dihedral group of order $2n$. We will denote this group by D_n. Consider the square with vertices labeled as in Figure 1.1 of this manual. (See the last page of this chapter for figures.) The way GAP denotes the element in D_4 that is a rotation by 90 degrees is $(1,2,3,4)$. This notation means vertex 1 goes to vertex 2, vertex 2 to 3, vertex 3 to 4 and 4 to 1. (Chapter 5 will give more details on this notation). Similarly, the horizontal reflection is denoted by $(1,2)(3,4)$. (See Figure 1.2.)

The command in GAP for the dihedral group D_n is `DihedralGroup(IsPermGroup,2n)`. For example to get D_4 we type:

```
gap> d4:= DihedralGroup(IsPermGroup,8);
Group([ (1,2,3,4), (2,4) ])
gap> Elements(d4);
[ (), (2,4), (1,2)(3,4), (1,2,3,4), (1,3), (1,3)(2,4), (1,4,3,2), (1,4)(2,3) ]
```

The command `Elements` listed the elements in the group. The identity is denoted by `()`. The command `Size` gives the number of elements in the group (that is, the order of the group).

```
gap> Size(d4);
8
```

The command `Size` is also useful to find the number of elements in a set. Elements can be multiplied (the operation is functional composition):

```
gap> (1,4)(2,3)*(2,4);
(1,2,3,4)
```

Careful: GAP multiplies these elements from left to right whereas many textbooks (including Gallian's) multiply these elements from right to left.

Exercises

1.1 Explain geometrically why a reflection followed by a reflection is a rotation. [Gallian, Chapter 1, Exercise 6] Using GAP take a reflection in D_4 and multiply it by a reflection in D_4. What rotation do you get?

1.2 Make a conjecture about what a rotation followed by a reflection is for **any** dihedral group. What about a reflection followed by a rotation? Test your conjecture by using GAP to compute the product of a reflection followed by a rotation for several pairs of reflections and rotations. You may want to draw a picture of the n-gon to help you determine which rotation or reflection you are getting.

1.3 Let r_1, r_2, r_3 represent rotations in D_n and let f_1, f_2, and f_3 represent reflections in D_n. Determine whether $r_1 r_2 f_1 r_3 f_2 f_3 r_3$ is a rotation or a reflection. [Gallian, Chapter 1, Exercise 10]

1.4 Using GAP, find elements A, B and C in D_5 such that $AB = BC$ but $A \neq C$. [Gallian, Chapter 1, Exercise 11]

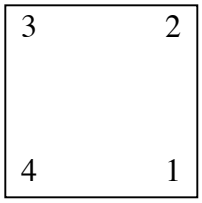

Figure 1.1

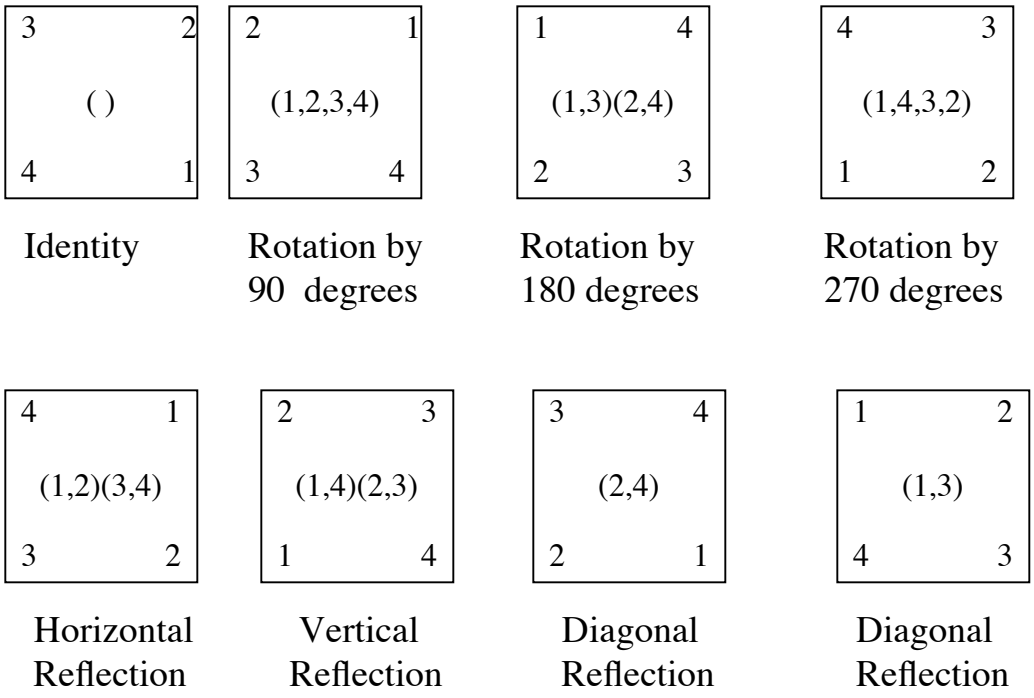

Figure 1.2