MT-A153-02 Final Exam Spring 2000

You may keep this page of questions. For this part of the exam, turn in your answers with all of your work on the green paper. You are not allowed to use Mathcad or a calculator for this part of the exam. Problems # 1–10 are worth 10 points each. Problems # 11-15 are worth 16 points each.

I. **(1)** Find $f'(x)$ if $f(x) = \cosh(5x)$.

(2) Find the Maclaurin series for $y = f(x) = \dfrac{1}{1-x^3}$. Express your final answer using summation notation.

II. Analyze and evaluate the following definite, indefinite, or improper integrals.

(3) $\displaystyle\int_0^1 \frac{2x}{x+4}\,dx$ **(4)** $\displaystyle\int_0^\infty \frac{x}{(x^2+4)^2}\,dx$

(5) $\displaystyle\int te^{5t}\,dt$ **(6)** $\displaystyle\int \sin 2\theta \cos 5\theta \, d\theta$

(7) $\displaystyle\int_{-1}^2 \frac{dx}{x^3}$ **(8)** $\displaystyle\int \frac{x^2-13x+36}{x^3-6x^2+9x}\,dx$

III. For each of the following series, determine whether the series converges or diverges. For an alternating series, distinguish between absolute and conditional convergence. State which test or tests you are using and show your work.

(9) $\displaystyle\sum_{k=1}^\infty \frac{k+1}{k^2+9}$ **(10)** $\displaystyle\sum_{k=0}^\infty \frac{(-1)^k 5^k}{k!}$

IV. The problems in section are worth 16 points each.

(11) Find the volume of the solid of revolution that is generated by revolving the region bounded by $y = 0$ and $y = \sin x$ between $x = 0$ and $x = \pi$ about the x-axis.

(12) Sketch a graph of the curve having polar equation $r = 4\sin\theta$ and identify this curve by name. Then find the slope of this curve at the point having polar coordinates $(2, \frac{5\pi}{6})$.

(13) Find $y = f(x)$ explicitly if $\frac{dy}{dx} = 6e^{-3x}\sqrt{y^2+1}$ and $f(0) = 0$.

(14) Find the interval of convergence for the power series

$$\sum_{k=0}^{\infty} \frac{k(x-7)^k}{(k+1)3^k}.$$

Determine whether the series converges or diverges at the endpoints. Show your work!

(15) The base of a solid is the region in the xy-plane bounded by $y = 3x$, $y = 0$ and $x = 2$. Every cross-section of the solid that is perpendicular to the y-axis is a square with a side in the base. Find the volume of this solid.

V. For these last two questions, it is intended that you **WILL** use Mathcad and/or a calculator. Do **NOT** work these two problems on the green paper. I have printed these two questions here in order that you might judge how much time you want to allow for them. When you have finished the problems in parts I – IV, turn them in on the green paper and receive a page of blue paper for these last two problems. They are worth 10 points each.

(16) Use Mathcad to find the antiderivative $\displaystyle\int x^2\sqrt{x^2+4x+13}\,dx$.

(17) Given the infinite series $S = \displaystyle\sum_{k=0}^{\infty} \frac{\sin^2 k}{\sqrt[3]{k+1}}$, use Mathcad to find

the values for the partial sums $S_{400}, S_{800}, S_{1600}, S_{3200}$ and S_{6400} of this series. Based upon these partial sums, what would you say about the convergence or divergence of this series? Reset the Mathcad format to display results with 6 digits of precision rather than its default of 3 digits of precision.