MATH143-01 Final Exam Spring 2009

You may keep this page of questions. Work the first 14 questions on the salmon paper. You are not allowed to use your calculator for this first part of the exam. After you have finished the first 14 questions, turn in all of the salmon paper and receive tan paper for the last two questions. Problems # 1–10 are worth 12 points each. I have marked the point value for the later problems and parts of problems on the exam.

I. Analyze and evaluate the following definite, indefinite, or improper integrals.

(1) $\displaystyle\int_0^{\pi/6} \sin(x)\cos(x)\,dx$ **(2)** $\displaystyle\int t^3 \sin(2t)\,dt$ **(3)** $\displaystyle\int \sqrt{4-x^2}\,dx$

(4) $\displaystyle\int_2^{\infty} \frac{dx}{(x+2)^2}$ **(5)** $\displaystyle\int \frac{dt}{t^2+9t+10}$ **(6)** $\displaystyle\int \frac{5x^2-12x-20}{x^3-4x}\,dx$

(7) $\displaystyle\int \sinh^{-1}(bw)\,dw$ where $b \neq 0$.

(8) Find the Maclaurin series for $y = f(x) = xe^{-3x}$. Express your final answer using summation notation.

(9) Solve the initial value problem: $\dfrac{dy}{dx} = 3e^{-y}x^2, \quad y(0) = 4.$

(10) Solve the initial value problem:

$$y'' - 3y' - 10y = 0, \quad y(0) = 5, \quad y'(0) = -3.$$

(11) 16 Points. Find the interval of convergence for the following power series. At the endpoints of the interval, either prove convergence of the series or else prove divergence.

$$\sum_{k=0}^{\infty} \frac{(-1)^k (x+5)^k}{(k^2+1)2^k}.$$

(12) 14 Points. Find the volume of the solid of revolution that is generated by revolving the region bounded by $x = 3$, $x = 5$, $y = 0$, and $y = \dfrac{1}{x^2}$ about the y-axis.

(13) 14 Points. Find the **mass** of the region in the xy-plane that is bounded by $y = x$ and $x = y^2 - 3y$ if the density δ at a point (x, y) within the region is given by $\delta = \sqrt{y}$.

(14) 10 Points. Let $f(x) = \begin{cases} 0 & \text{if } x < 0 \\ kxe^{-x^2} & \text{if } x \geq 0 \end{cases}$ For what value of k will $f(x)$ be a probability density function?

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(15) 12 Points. Using the table for $f(x)$ below, find the numerical approximations T_6 and S_6 for the integral $\displaystyle\int_1^4 f(x)\,dx$.

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	5.17	4.83	4.92	5.58	5.93	6.22	6.41

(16) 14 Points. A demographer studying the population of a certain small country uses the logistic model

$$P = \frac{L}{1 + Ae^{-kt}}$$

The population of the country was 2.403 million at the beginning of 1970. From a careful analysis of annual population studies, the demographer estimates that the inflection point for the logistic curve occurred at the beginning of 2004 when the population was 3.145 million. Evaluate the parameters for the logistic model. Using these, what population does the logistic model predict at the beginning of 2030?