MATH143-01 Final Exam Spring 2008

You may keep this page of questions. Work the first 12 questions on the gold paper. You are not allowed to use your calculator for this first part of the exam. After you have finished the first 12 questions, turn in all of the gold paper and receive blue paper for the last two questions. Problems # 1–8 are worth 12 points each. I have marked the point value for the later problems and parts of problems on the exam.

I. Analyze and evaluate the following definite, indefinite, or improper integrals.

(1) $\displaystyle\int \cos^5\theta \sin\theta \, d\theta$ **(2)** $\displaystyle\int_0^\infty e^{-7x}\, dx$ **(3)** $\displaystyle\int_0^1 \frac{dx}{\sqrt{4-x^2}}$

(4) $\displaystyle\int_0^\pi \frac{d\theta}{\cos^2\theta}$ **(5)** $\displaystyle\int e^{-2t}\sin 3t\, dt$ **(6)** $\displaystyle\int \frac{x^2-5x-13}{(x-4)(x^2+1)}\, dx$

(7) $\displaystyle\int \frac{dx}{\sqrt{x^2+a^2}}$ where $a > 0$.

(8) Find the Maclaurin series for $y = f(x) = \dfrac{x^3}{1+x^3}$. Express your final answer using summation notation.

(9) 18 Points. Solve the initial value problem: $\dfrac{dy}{dx} = \dfrac{\sin(3x)}{y^2}, \quad y(0) = 2.$

(10) 18 Points. Find the interval of convergence for the following power series. At the endpoints of the interval, either prove convergence of the series or else prove divergence.

$$\sum_{k=0}^{\infty} \frac{(x-3)^k}{(2k+1)5^k}.$$

(11) 18 Points. The base of a solid is the region in the xy-plane bounded by $x = 2$ and $2x = y^2$. Every cross section of the solid perpendicular to the y-axis is a square with a side in the base. Find the volume of the solid.

(12) 18 Points. Find the area of the region that is inside the cardioid $r = 1 - \cos\theta$.

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(13)(a) 12 Points. Find the partial sums S_{100}, S_{200} and S_{400} for the series $\displaystyle\sum_{k=1}^{\infty} \frac{\ln(1.12 + \sin(k))}{k}$.

(b) 4 Points. Based upon these calculations, do you expect that this series converges or that it diverges?

(14) 16 Points. A demographer studying the population of a certain small country uses the logistic model

$$P = \frac{L}{1 + Ae^{-kt}}$$

The population of the country was 1.814 million at the beginning of 1990. From a careful analysis of annual population studies, the demographer estimates that the inflection point for the logistic curve occurred at the beginning of 2001 when the population was 2.409 million. Evaluate the parameters for the logistic model. Using these, what population does the logistic model predict at the beginning of 2025?