MATH143-01 Exam Three Spring 2008

You may keep this page of questions. Turn in your answers with all of your work on the pink paper and the yellow paper. You are NOT allowed to use your calculator on the first five questions. Answer these five questions on the pink paper. When you have completed these five questions, turn in all of the pink paper and receive yellow paper to use on the last three questions. The last three questions are printed on the back of this page. You ARE allowed to use your calculator on the last three questions and you will need your calculator for some parts of the last three questions.

I. (1) 12 Points. Find the Maclaurin series for $f(x) = \dfrac{x^3}{1+x}$. You are expected to use a known power series and to express your final answer using summation notation.

(2) 10 Points. Find an **exact** value for the sum of the following series by recognizing the series as a Maclaurin series evaluated at a particular value of x.

$$\frac{\pi}{3} - \frac{(\frac{\pi}{3})^3}{3!} + \frac{(\frac{\pi}{3})^5}{5!} - \frac{(\frac{\pi}{3})^7}{7!} + \frac{(\frac{\pi}{3})^9}{9!} - \frac{(\frac{\pi}{3})^{11}}{11!} + \cdots$$

(3) 12 Points. Either **prove** that the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{3n^2+7}$ converges or else **prove** that this series diverges.

(4) 12 Points. Either **prove** that the series $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{(2n+1)!}$ converges or else **prove** that this series diverges.

(5) 18 Points. Find the interval of convergence, including endpoint behavior, for the power series $\displaystyle\sum_{k=1}^{\infty} \frac{2^k(x-3)^k}{k^2+5}$.

You will need your calculator for **these** three questions. Turn in all of your work on the first five questions and any extra pink paper. You will receive yellow paper to use for these last three questions.

II. (6) 12 Points. Find the partial sums S_{100}, S_{200} and S_{400} for the series $\displaystyle\sum_{k=1}^{\infty} \frac{\cos(5k+2)}{k\ln(k+3)}$. Based upon these calculations, do you expect that this series converges or that it diverges?

(7) Suppose that g is a function which has continuous derivatives, and that $g(7) = -5$, $g'(7) = 3$, $g''(7) = -4$ and $g'''(7) = 24$.

(a) 8 Points. Find the degree 3 Taylor polynomial for g near 7.

(b) 4 Points. Use the degree 3 Taylor polynomial for g near 7 to approximate $g(7.0026)$.

(8) 12 Points. If you save \$200.00 at the end of every month for 60 years and invest this money at 4.5% nominal annual interest compounded monthly, how much money will you have at the end of 60 years?