MT-A143-02 Make-Up Exam Two Spring 2003

You may keep this page of questions. Turn in your answers with all of your work on the tan paper and the orchid paper. Work the three questions on this side of this page **without** your calculator. After you have finished these three questions, turn them in and receive orchid paper to use for the three questions on the back of this page. You **will** need to use your calculator on some parts, but not all parts, of the last three questions.

(1) 16 Points. The base of a solid is the region in the xy-plane that is between $x = 0$ and $x = \frac{\pi}{2}$ and is bounded above and below by $y = \sin x$ and $y = 0$. Every cross-section of this solid perpendicular to the x-axis is a semicircle with its diameter in the base. Find the volume of this solid.

(2) 14 Points. Analyze and evaluate $\displaystyle\int_1^8 \frac{1}{\sqrt[3]{8-t}}\, dt$.

(3) 18 Points. Find the mass M, the moment M_x with respect to the x-axis, the moment M_y with respect to the y-axis, and the center of mass, $(\bar{x}, \bar{y})$, for the plane region in the first quadrant bounded by $x = 3, y = 6$ and $y = \frac{6}{x}$ if this region has density $\delta = y$ at a point (x, y) within the region.

(4) 16 Points. Find the kinetic energy of a rod of mass 61.50 kg and length 10.00 m rotating about an axis perpendicular to the rod at its midpoint, with an angular velocity of 4.800 radians per second. Use the fact that the kinetic energy, E_K, of a particle of mass m moving at a speed v is given by $E_K = \frac{1}{2}mv^2$.

(5) 16 Points. **(a)** Set up a definite integral that gives the arc length of the curve defined parametrically by

$$x = te^{3t}, \quad y = \cos(\pi t) \quad \text{for} \quad 0 \leq t \leq 1.$$

(b) Use your calculator to find the Simpson's Rule approximation S_{50} for this integral for arc length.

(6) 20 Points. In Section 8.7, I assigned exercise 7 on page 396 which begins as follows:

> `Consider a group of people who have received treatment for a disease such as cancer. Let` t `be the` *survival time,* `the number of years a person lives after receiving treatment. The density function giving the distribution of` t `is` $p(t) = Ce^{-Ct}$ `for some positive constant` C.

(a) A more careful description of the density function above could give $p(t)$ as the piecewise defined function

$$p(t) = \begin{cases} 0 & \text{if } t < 0 \\ Ce^{-Ct} & \text{if } t \geq 0 \end{cases}$$

Explain why it is clear from the textbook's description that $p(t) = 0$ if $t < 0$.

(b) What is the practical meaning for the cumulative distribution function $P(t) = \int_0^t p(x)\,dx$?

(c) The survival function, $S(t)$, is the probability that a randomly selected person survives for at least t years. Find $S(t)$.

(d) Suppose that a patient has a 65% probability of surviving at least three years. Find C.

(e) Find the mean survival time, in years, for survival after treatment modeled by this density function.