

You may keep this page of questions. Work the first 12 questions on the canary paper. You are not allowed to use your calculator for this first part of the exam. After you have finished the first 12 questions, turn in all of the canary paper and receive green paper for the last three questions. Problems # 1–10 are worth 12 points each. I have marked the point value for the later problems and parts of problems on the exam.

I. Analyze and evaluate the following definite, indefinite, or improper integrals.

$$(1) \int_0^3 \frac{x \, dx}{x^2 + 16} \quad (2) \int_1^5 \frac{dx}{(x-4)^2} \quad (3) \int \frac{5x-32}{x^3+16x} dx$$

$$(4) \int_0^\infty e^{-4x} dx \quad (5) \int \sin(3x) \sin(5x) dx \quad (6) \int \frac{x^3 dx}{\sqrt{25-x^2}}$$

II. (7) Find $f'(t)$ if $f(t) = \int_\pi^t e^{\sqrt{4+\sin \theta}} d\theta$.

(8) Find the Maclaurin series for $y = f(x) = x \cosh(3x)$. Express your final answer using summation notation. Hint: $\cosh z = \frac{1}{2}(e^z + e^{-z})$.

(9) Solve the initial value problem: $\frac{dy}{dt} = 4y^2 \sinh(2t), \quad y(0) = 3$.

(10) Find the interval of convergence, including endpoint behavior, for the power series

$$\sum_{k=1}^{\infty} \frac{(x-3)^k}{k^2 2^k}.$$

(11) 16 Points. Find the volume of the solid of revolution that is generated by revolving the region bounded by $y = x$ and $x = (y - 2)^2$ about the x -axis.

(12) 18 Points. Suppose that x measures that time (in minutes) it takes for a student to complete a quiz. Assume that all students are done in 6 minutes and the density function for x is given by

$$p(x) = \begin{cases} k(6x^2 - x^3) & \text{if } 0 < x < 6 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the value of k required for $p(x)$ to be a density function.
 - (b) What fraction of students finish the quiz in 5 minutes or less?
 - (c) What is the mean time for students to complete the quiz?
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(13) 12 Points. Find the partial sums S_{40} , S_{80} and S_{160} for the series

$$\sum_{k=1}^{\infty} \frac{k \cos(k)}{k^2 + 1}.$$

Based upon these calculations, do you expect that this series converges or that it diverges?

(14a) 7 Points. Set up a definite integral for the arc length of the curve which is defined parametrically by $x = e^{-t}$, $y = t \sin(\pi t)$ for $0 \leq t \leq 2$.

(14b) 7 Points. Find the trapezoidal approximation, T_{10} , for the integral in part (14a).

(15) In this problem, you are to use your calculator and Euler's method to find approximate solutions of the initial value problem

$$\frac{dy}{dx} = \sin(xy) + 1 - x^2, \quad y(1.0) = 2.0.$$

For parts (a) and (b) below, you should round y values to the nearest thousandth. On part (b), I am expecting you to write down every tenth value for x and for y .

(15a) 8 Points. Use your calculator and Euler's method with step size $h = 0.1$ and five Euler steps to approximate the solution of the initial value problem.

(15b) 8 Points. Use your calculator and Euler's method with step size $h = 0.01$ and fifty Euler steps to approximate the solution of the initial value problem.

(15c) 4 Points. Based upon parts (a) and (b), what, if anything, might you say about the accuracy of these approximate solutions.