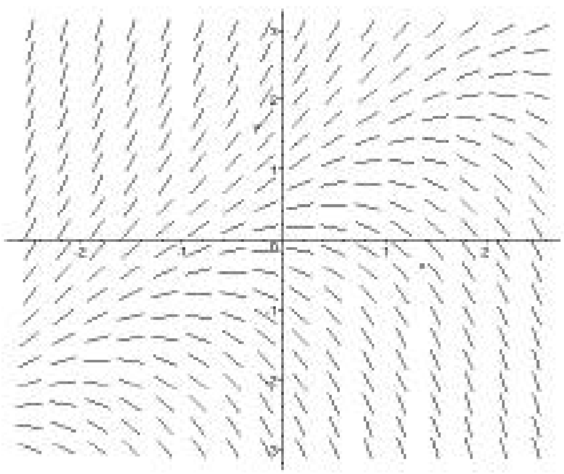
MT-A143-01 Exam Four Spring 2003

You may keep this page of questions. Turn in your answers with all of your work on the colored paper. You are not allowed to use a calculator for the first five questions, but you will need your calculator for the last question.

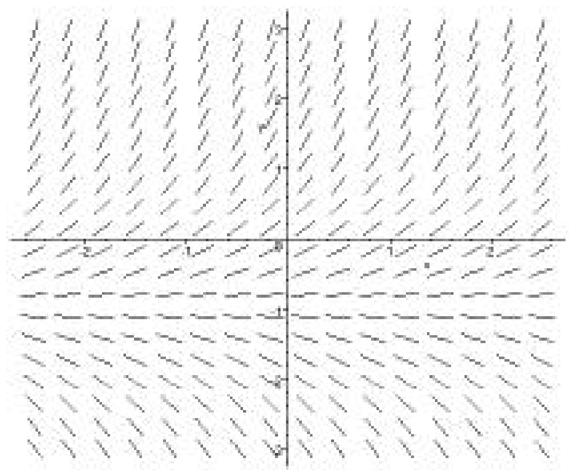
(1) 16 Points. Match each of the following differential equations with one of the slopefields below.

(a) $\dfrac{dy}{dx} = -x^2$ **(b)** $\dfrac{dy}{dx} = y - x$ **(c)** $\dfrac{dy}{dx} = \sin[2(y-x)]$ **(d)** $\dfrac{dy}{dx} = y + 1$

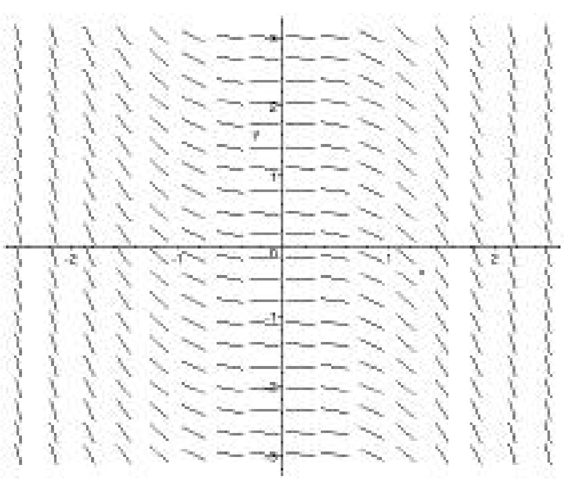
(I)



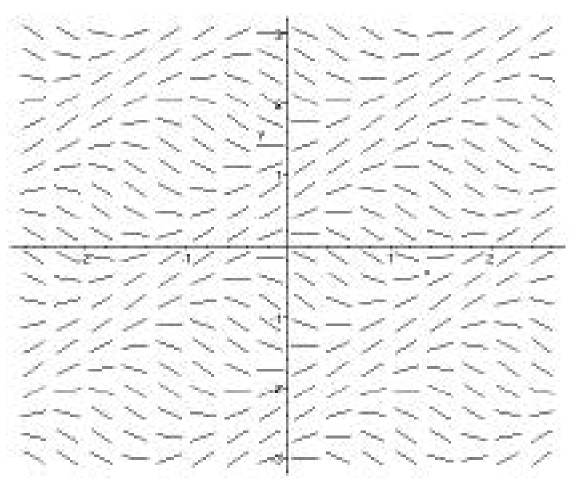
(II)



(III)



(IV)



(2) 12 Points. On the tan page, I have printed for you a slope field for the differential equation $\frac{dy}{dx} = y + 2x + 1$. On this slope field sketch a graph of the solution of the initial value problem

$$\frac{dy}{dx} = y + 2x + 1, \quad y(-1.5) = 0.$$

(3) 12 Points. Suppose that $Q = Ce^{kt}$ is a solution of the differential equation $\frac{dQ}{dt} - 0.17Q = 0$. What (if anything) does this tell you about the values of C and k?

(4) 20 Points. Solve the initial value problem

$$\frac{dy}{dt} = \frac{\sin(3t)}{y^2}, \quad y(0) = 3.$$

(5) **(a)** 8 Points. As modeling assumptions, assume that dead leaves accumulate on the ground in a forest at the rate of 4 grams per square centimeter per year while at the same time these fallen leaves decompose at a continuous rate of 80% per year. Write a differential equation for the total mass (per square centimeter), M, of dead leaves on the ground at time t.

(b) 8 Points. Find an explicit general solution for your differential equation in part **(a)**.

(c) 4 Points. Discuss the nature of any equilibrium solutions for this differential equation.

(d) 4 Points. Do you think that this model adequately describes the accumulation of dead leaves in the forest? **Explain!** Why might the **location** of the forest be significant?

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(6) 16 Points. Use your calculator and Euler's method with step size $h = 0.1$ and seven Euler steps to approximate the solution of the initial value problem

$$\frac{dy}{dx} = 2x - y^2, \quad y(1.5) = -1.$$

Round the y values to the nearest thousandth.

Slope Field Page

(2) 12 Points. The slope field below is a slope field for the differential equation $\frac{dy}{dx} = y + 2x + 1$. On this slope field sketch a graph of the solution of the initial value problem

$$\frac{dy}{dx} = y + 2x + 1, \quad y(-1.5) = 0.0.$$