MT-A143-02 Exam Three Spring 2003

You may keep this page of questions. Turn in your answers with all of your work on the green paper and the yellow paper. You are NOT allowed to use your calculator on the first six questions. Answer these six questions on the green paper. When you have completed these six questions, turn in all of the green paper and receive yellow paper to use on the last two questions. You ARE allowed to use your calculator on the last two questions and you will need your calculator for some parts of the last two questions.

I. (1) 12 Points. Find the Maclaurin series for $f(x) = x^2 e^{-3x}$. You are expected to use a known power series and to express your final answer using summation notation.

(2) 10 Points. Find an **exact** value for the sum of the following series by recognizing the series as a Maclaurin series evaluated at a particular value of x.

$$\frac{1}{3} - \frac{(\frac{1}{3})^2}{2} + \frac{(\frac{1}{3})^3}{3} - \frac{(\frac{1}{3})^4}{4} + \frac{(\frac{1}{3})^5}{5} - \frac{(\frac{1}{3})^6}{6} + \cdots$$

(3) 14 Points. Find the degree 3 Taylor polynomial for $f(x) = \sqrt[3]{4+3x}$ about $a = -1$.

(4) 16 Points. Find the interval of convergence, including endpoint behavior, for the power series

$$\sum_{k=1}^{\infty} (-1)^k \frac{(x-2)^k}{(3k+1)5^k}.$$

II. For each of the following series, either **prove** that the series converges or else **prove** that the series diverges. For an alternating series, distinguish between absolute and conditional convergence. State which test or tests you are using and show your work. These two questions are worth 14 points each.

(5) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k 5^k}{k(2k-1)!}$ **(6)** $\displaystyle\sum_{k=0}^{\infty} \frac{10}{k^2+2k+5}$

III.

(7) 10 Points. Find the partial sums S_{20}, S_{40} and S_{80} for the series $\displaystyle\sum_{k=1}^{\infty} \frac{\sin(3k+2)}{k\ln(k+1)}$

Based upon these calculations, do you expect that this series converges or that it diverges?

(8) 10 Points. If you save \$250.00 at the end of every month for 45 years and invest this money at 3.6% nominal annual interest compounded monthly, how much money will you have at the end of 45 years?