MT-A143-01 Make-Up Exam Three Spring 2002

You may keep this page of questions. Turn in your answers with all of your work on the blue paper and tan paper. You are **NOT** allowed to use calculators on questions #1 – 6. Work these questions on the blue paper. After you have finished these first six questions, turn in the first part of the exam and receive a page of tan paper to use for the last two questions. You will need to use your calculator for parts of the last two questions.

I. (1) 12 Points. Find all values of x for which the series $\sum_{j=3}^{\infty} x^{4j+1}$ converges. Then state the limit of the series as a simple expression involving x.

(2) 12 Points. Find the Maclaurin series for $f'(x)$ if $f(x) = x^2 \arctan x$. Observe that this question asks for the series for $f'(x)$ rather than for $f(x)$. You are expected to use a known power series and to express your final answer using summation notation.

(3) 14 Points. Suppose that the power series $\sum_{k=0}^{\infty} a_k(x-2)^k$ converges if $x = 6$ and diverges if $x = -5$. Indicate which of the following statements *must* be true, which *may* be true and which *cannot* be true. Justify your answers.

(a) The power series converges if $x = -2$.
(b) The power series diverges if $x = 11$.
(c) The power series converges if $x = 6.25$.
(d) The power series diverges if $x = -4$.
(e) The power series converges if $x = 10$.
(f) The power series diverges if $x = -1$.
(g) The power series converges absolutely if $x = 3$.

(4) 14 Points. Compute a useful expression for the partial sum S_n for the series $\sum_{k=1}^{\infty} \left(\int_k^{k+1} \frac{dx}{x^3} \right)$. Use this expression to determine whether the series converges or diverges. If the series converges, find its limit.

II. For each of the following series, either **prove** that the series converges or else **prove** that the series diverges. For an alternating series, distinguish between absolute and conditional convergence. State which test or tests you are using and show your work. These two questions are worth 12 points each.

(5) $\sum_{k=0}^{\infty} \frac{(-1)^k \sqrt{k}}{k^2+9}$ **(6)** $\sum_{k=1}^{\infty} \frac{7^k}{k^5 6^k}$

III.

(7) 14 Points. Find the partial sums S_{400} and S_{401} for the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k}}$.

Based upon these calculations, find upper and lower bounds for the sum of the series, find an approximation for the sum of the series, and discuss the maximal possible error in this approximation for the sum.

(8) 10 Points. If you save \$250.00 at the end of every month for 45 years and invest this money at 5.4% nominal annual interest compounded monthly, how much money will you have at the end of 45 years?