MT-A143-01 Final Exam Spring 2002

You may keep this page of questions. Work the first 14 questions on the blue paper and the white slopefield page. You are not allowed to use your calculator for this first part of the exam. After you have finished the first 14 questions, turn in the blue paper and the slopefield page and receive pink paper for the last three questions. Problems # 1–10 are worth 10 points each. Problems # 11–14 are worth 16 points each. Problems # 15–17 are worth 12 points each.

I. Analyze and evaluate the following definite, indefinite, or improper integrals.

(1) $\int \sin^3\theta \, d\theta$ **(2)** $\int_0^\infty e^{-5x}\,dx$ **(3)** $\int \sin^{-1}(3t)\,dt$

(4) $\int_0^2 \frac{x}{(x^2+5)^2}\,dx$ **(5)** $\int_{-1}^1 \frac{dx}{\sqrt[3]{x^5}}$

(6) $\int \frac{dx}{\sqrt{x^2+25}}$ **(7)** $\int \frac{x^2+7x+27}{x(x+3)^2}\,dx$

(8) $\int (mx+b)^{-1}\,dx$ where $m > 0$ and $b > 0$.

II. (9) On the **white** page, I have printed for you a slope field for the differential equation $\frac{dy}{dx} = x\sin(5y-2x+1)$. On this slope field sketch a graph of the solution of the initial value problem

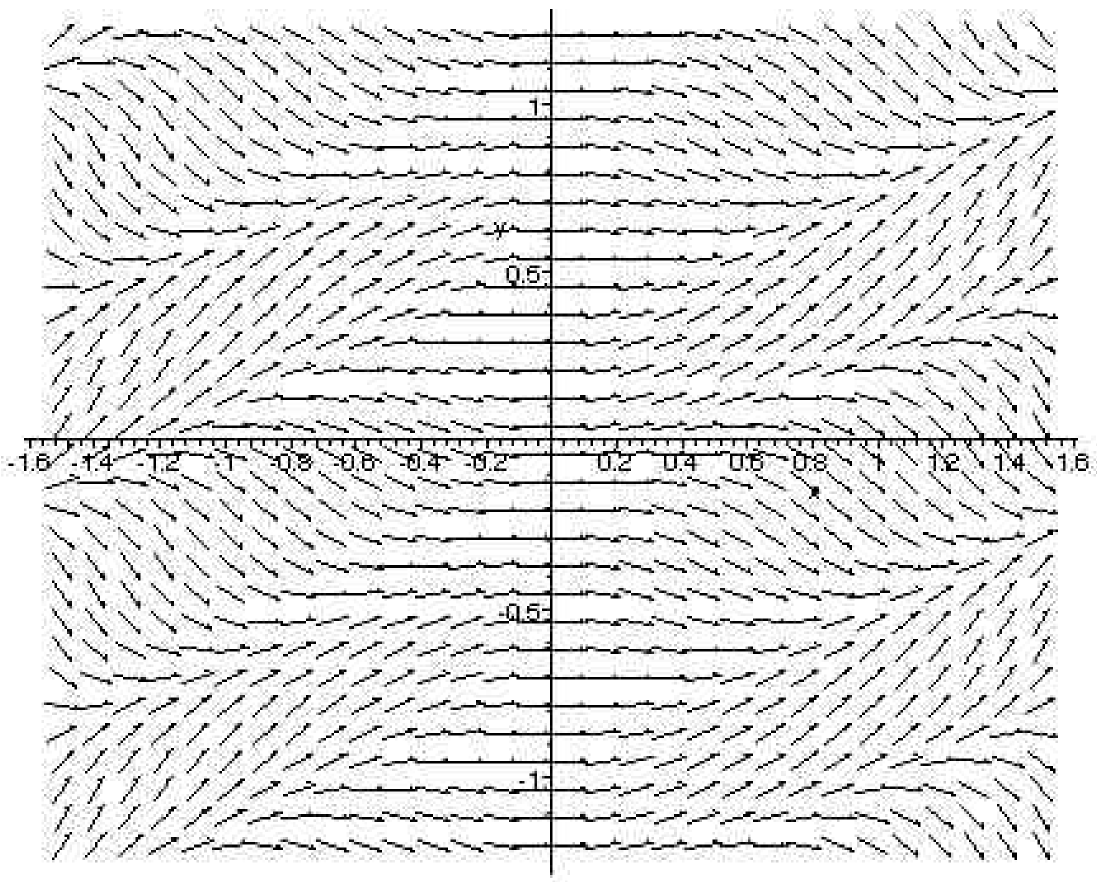
$$\frac{dy}{dx} = x\sin(5y-2x+1) \quad y(-1.0) = 0.5.$$

(10) Find the Maclaurin series for $y = f(x) = \frac{x}{1-x^4}$. Express your final answer using summation notation.

Slope Field Page

(9) The slope field below is a slope field for the differential equation $\frac{dy}{dx} = x\sin(5y - 2x + 1)$. On this slope field sketch a graph of the solution of the initial value problem

$$\frac{dy}{dx} = x\sin(5y - 2x + 1), \quad y(-1.0) = 0.5.$$



(11) Find the volume of the solid of revolution that is generated by revolving the region bounded by $y = x$ and $y = 3x - x^2$ about the y-axis.

(12) Find the interval of convergence, including endpoint behavior, for the power series

$$\sum_{k=1}^{\infty} \frac{(x+2)^k}{(k+5)3^k}.$$

(13) Evaluate $\displaystyle\lim_{x \to 0} \frac{1 + 3x - e^{3x}}{1 - \cos(5x)}$.

(14) Sketch a graph of the curve having polar equation $r = 4\sin\theta$.

Then find the slope of this curve at the point $(r, \theta) = (2, \frac{5\pi}{6})$.

(15a) 8 Points. Set up a definite integral for the arc length of the curve which is defined parametrically by $x = t^2$, $y = \cos(2t)$ for $0 \le t \le \pi$.

(15b) 4 Points. Use the numerical integration on your calculator (the fnInt function) to find a numerical approximation for the integral in part **(15a)**.

(16) Suppose that $I = \int_2^5 f(x)\,dx$ and that on the interval $[2, 5]$ we have $|f'(x)| \le 4.0$, $|f''(x)| \le 11.5$, $|f'''(x)| \le 23.0$ and $|f^{(4)}(x)| \le 36.0$. Find a value of n which is large enough to guarantee that $|I - S_n| \le 0.00005$ where S_n is the n^{th} Simpson's rule approximation for I. Show your work!

(17) Find the partial sums S_{20} and S_{21} for the series $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{1 + \sqrt{k}}$.

Based upon these calculations, find upper and lower bounds for the sum of the series, find an approximation for the sum of the series, and discuss the maximal possible error in this approximation for the sum.