MT-A143-01 Exam Four Spring 2002

You may keep this page of questions. Turn in your answers with all of your work on the colored paper. You are not allowed to use a calculator for the first six questions, but you will need your calculator for the last question.

(1) 16 Points. Change the polar equation $r = 6\cos\theta$ to an equivalent equation in x and y. Then sketch the graph of the curve given by these equations.

(2) 10 Points. On the green page, I have printed for you a slope field for the differential equation $\frac{dy}{dx} = 2x^2 - 3y^2 - 1$. On this slope field sketch a graph of the solution of the initial value problem

$$\frac{dy}{dx} = 2x^2 - 3y^2 - 1, \quad y(0.5) = 2.0.$$

(3a) 12 Points. If a particle moves along a curve with position given by $x = 4\sin(t^3),\ y = 4\cos(t^3)$, find the speed of the particle when $t = 3$.

(3b) 6 Points. Briefly describe, **in words,** the curve that this particle follows and how the particle moves along this curve for $t \geq 0$.

(4) 16 Points. Find the area of the region that is contained in one leaf of the rose $r = 7\cos(2\theta)$.

(5) 12 Points. For each of the following functions, decide whether or not the function is a solution of the differential equation $t\frac{dy}{dt} + 3y = 12$. SHOW YOUR WORK!

a) $y = f(t) = 4 + t^{-3}$ **b)** $y = g(t) = 2 + t^{-3}$ **c)** $y = h(t) = 4 - t^{-3}$

(6) 16 Points. Solve the initial value problem

$$\frac{dy}{dt} = 2y^2 e^{-3t}, \quad y(0) = 3.$$

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(7) 12 Points. Use your calculator and Euler's method with step size $h = 0.05$ and 5 Euler steps to approximate the solution of the initial value problem

$$\frac{dy}{dt} = \sqrt{y + 4t + 1}, \quad y(1) = 4.$$

Round the y values to the nearest thousandth.

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Slope Field Page

(2) 10 Points. The slope field below is a slope field for the differential equation $\frac{dy}{dx} = 2x^2 - 3y^2 - 1$. On this slope field sketch a graph of the solution of the initial value problem

$$\frac{dy}{dx} = 2x^2 - 3y^2 - 1, \quad y(0.5) = 2.0.$$