MT-A143-01 Final Exam Fall 2004

You may keep this page of questions. Work the first 12 questions on the pink paper and question 13 on the yellow slopefield page. You are not allowed to use your calculator for this first part of the exam. After you have finished the first 13 questions, turn in all of the pink paper and the yellow slopefield page and receive green paper for the last question. Problems # 1–8 are worth 12 points each. I have marked the point value for the later problems and parts of problems on the exam.

I. Analyze and evaluate the following definite, indefinite, or improper integrals.

(1) $\int \sin^3\theta\cos\theta\, d\theta$ **(2)** $\int_0^\infty xe^{-3x^2}\, dx$ **(3)** $\int \frac{x^2+13x-20}{x^3-4x^2} dx$

(4) $\int_0^9 (x-1)^{-4/3}\, dx$ **(5)** $\int_0^{\pi/2} x\cos x\, dx$ **(6)** $\int \frac{dx}{(x^2+25)^2}$

(7) For the series below, find the partial sum, S_n, and the sum, S:

$$3 - \frac{3}{4} + \frac{3}{16} - \frac{3}{64} + \frac{3}{256} - \frac{3}{1024} + \cdots$$

(8) Find the Maclaurin series for $y = f(x) = xe^{-x^2}$. Express your final answer using summation notation.

(9) 16 Points. Solve the IVP: $\frac{dy}{dt} = 12y^2\sin(3t), \quad y(\pi) = 1/3.$

(10) 16 Points. Find the interval of convergence, including endpoint behavior, for the power series

$$\sum_{k=1}^{\infty} \frac{(x-1)^k}{(k^2+9)4^k}.$$

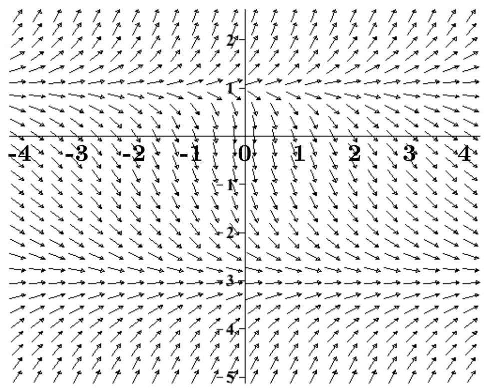
(11) 20 Points. Find the volume of the solid of revolution that is generated by revolving the region bounded by $y = x - 1$ and $x = (y - 1)^2$ about the x-axis.

(12) 16 Points. Find the degree 2 Taylor polynomial for $f(x) = \tan(3x)$ about $a = \pi/12$.

Use the yellow slopefield page for this question!

I am printing a very brief version of the question here for your records.

(13) 16 Points. For the differential equation $\frac{dy}{dx} = \frac{y^2+2y-3}{\sqrt{x^2+y^2}}$, sketch the solution of the IVP's with **(a)** $y(-3) = 0.5$, **(b)** $y(-3) = -4$ and **(c)** identify and describe any equilibrium solutions of this differential equation.



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(14) 20 Points. A demographer studying the population of a certain small country uses the logistic model

$$P = \frac{L}{1 + Ae^{-kt}}$$

The population of the country was 2.023 million at the beginning of 1970. From a careful analysis of annual population studies, the demographer estimates that the inflection point for the logistic curve occurred at the beginning of 1986 when the population was 3.230 million. Evaluate the parameters for the logistic model. Using these, what population does the logistic model predict at the beginning of 2010?

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Slope Field Page

(13) The slope field below is a slope field for the differential equation $\frac{dy}{dx} = \frac{y^2+2y-3}{\sqrt{x^2+y^2}}$.

(a) 5 Points. On this slope field sketch a graph of the solution of the initial value problem

$$\frac{dy}{dx} = \frac{y^2+2y-3}{\sqrt{x^2+y^2}}, \quad y(-3.0) = 0.5.$$

(b) 5 Points. On this slope field sketch a graph of the solution of the initial value problem

$$\frac{dy}{dx} = \frac{y^2+2y-3}{\sqrt{x^2+y^2}}, \quad y(-3.0) = -4.0.$$

(c) 6 Points. Does this differential equation have any equilibrium solutions? Describe every such equilibrium solution and identify it as *stable* or *unstable*.