Section 15 The Product Topology on X x Y

Definition: Let X and Y be topological spaces.
The **product topology** on X x Y is the topology having as a basis the collection B of all sets of the form U x V, Where U is open in X and V is open in Y.

Theorem 15.1 If B is a basis for the topology of X and $\mathscr{C}$ is the basis for a topology of Y, then $\mathscr{D} = \{B \times C \mid B \in \mathscr{B} \text{ and } C \in \mathscr{C}\}$ is a basis for the topology of X x Y.

Definition: The projections onto the first and second factors of X x Y are:
$\pi_1 : X \times Y \to X$ is defined by $\pi_1(x, y) = x$
$\pi_2 : X \times Y \to Y$ is defined by $\pi_2(x, y) = y$

Theorem 15.2 The collection $S = \left\{\pi_1^{-1}(U) \mid U \text{ open in X}\right\} \cup \left\{\pi_2^{-1}(V) \mid V \text{ open in Y}\right\}$
Is a sub-basis for the product topology on X x Y.

Look at some examples ….

X and Y are the real line, look at different topologies on each.
X is the real line, Y is a finite set. Consider different topologies.