Section 14 The Order Topology

Definition: Recall that **an order relation** "$<$" is a relation with the following properties:
1. If $x \neq y$, then either $x < y$ or $y < x$ (Comparability)
2. If $x < y$ then $x \neq y$ (Non-reflexivity)
3. If $x < y$ and $y < z$, then $x < z$ (Transitivity)

(pg 24 – 25)

Definition: Let X be a set with a simple order relation; Assume X has more than one element.
A basis B for the order topology on X is given by the collection of all sets of these types:
1. All open intervals (a,b) in X
2. All intervals of the form $[a_0,b)$, where a_0 is the smallest element (if any) of X
3. All intervals of the form $[a,b_0)$, where b_0 is the largest element (if any) of X

EX 1. The standard topology on the real numbers is the order topology derived from the usual order on the real numbers.

EX2. Order topology on R x R induced by the dictionary order. What do basis elements look like?

Definition: **open and closed rays**:

$(a,\infty) = \{x \mid x > a\}$

$(-\infty,a) = \{x \mid x < a\}$

$[a,\infty) = \{x \mid x \geq a\}$

$(-\infty,a] = \{x \mid x \leq a\}$