

Section 12 Topological Spaces

Definition:

A *topological space* is as a set X together with a collection T of subsets of X satisfying the following axioms:

1. The empty set and X are in T .
2. The union of any collection of sets in T is also in T .
3. The intersection of any pair of sets in T is also in T .

The collection T is called a *topology* on X .

The sets in T are the open sets, and their complements in X are the closed sets

Comparing Topologies:

When every set in a topology T_1 is also in a topology T_2 ($T_1 \subset T_2$), we say that T_2 is finer than T_1 , and T_1 is coarser than T_2 .

Look at:

1. $X = \{a, b, c\}$. Describe some topologies on X .

For X any set:

2. The discrete topology on X is the collection of *all* subsets.
3. The Indiscrete or Trivial topology on X is the collection T consisting only of the empty set and X .
4. Finite Complement Topology: $T_F = \{U \mid X - U \text{ is finite or all of } X\}$. Show it's a topology.