

Exam 2 Math 135 Discrete Math
Monday, November 13, 2006

1. [10 pts] Draw a Venn diagram and shade the given set for the following:

a. $A \cap \overline{B}$

b. $B \cap \overline{(C \cup A)}$

2. [15 pts] a. Define what a 1-1 function is.

b. Define what an onto function is.

c. Determine if the function $f(x) = 2^x + 3$ is one-to-one and onto. Explain why.

3. [25 pts] a. Define $f(n) = O(g(n))$. What is this notation called?

b. Define $f(n) = \Omega(g(n))$. What is this notation called?

c. Define $f(n) = \Theta(g(n))$. What is this notation called?

d. Suppose an algorithm takes $35n^3 + 2n^2 + n + 200$ units of time to execute.

Find the theta notation in terms of n for this algorithm.

4. [15 pts] Given the sequence

$$q_1 = 4, q_2 = 8, q_3 = 10, q_4 = 12, q_5 = 15$$

Find $\sum_{k=1}^4 q_k$

Circle the correct answers:

Is the sequence q increasing? Yes No

Is the sequence q non-increasing? Yes No

Is the sequence q decreasing? Yes No

Is the sequence q non-decreasing? Yes No

5. [15 pts] Write an algorithm that returns the largest and second largest values in the sequence s_1, s_2, \dots, s_n . Assume that $n > 1$ and that the values in the sequence are all distinct.

Shuffle

This algorithm shuffles the values in the sequence a_1, a_2, \dots, a_n

Input: a, n

Output: a (shuffled)

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Shuffle( $a, n$ ) {
    for  $i=1$  to  $n-1$ 
        swap( $a_i, a_{\text{rand}(i,n)}$ )}

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6. [10 pts] Trace the shuffle algorithm for the input 34 57 99 75 200

Assume that the values of rand are:

$\text{rand}(1,5) = 2$ $\text{rand}(2,5) = 5$

$\text{rand}(3,5) = 4$ $\text{rand}(4,5) = 3$

Algorithm 4.4.6 Robot Walking

This algorithm computes the function defined by

$$\text{walk}(n) = \begin{cases} 1 & n = 1 \\ 2 & n = 2 \\ \text{walk}(n-1) + \text{walk}(n-2) & n > 2 \end{cases}$$

Input: n

Output: $\text{walk}(n)$

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1 Walk( $n$ ) {
2     If ( $n == 1$  or  $n == 2$ )
3         Return  $n$ 
4     Return  $\text{walk}(n-1) + \text{walk}(n-2)$ 
5 }

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7. [10 pts] Trace algorithm 4.4.6 for $n = 4$.

8. [10 pts] Prove that algorithm 4.4.6 is correct.

9. [10 pts] Write another robot walking algorithm if the robot can take steps of length 1, 2 and 3.