

**Exam 1, MATH 135 Discrete Mathematics. Name: \_\_\_\_\_**  
**Spring 2008 Dr. A. Bart.**

1. [15 pts] Write the truth table for the proposition  $(p \wedge q) \vee (\neg p \vee q)$   
(The truth table is enough, no further work needs to be shown.)

2. [25 pts] Determine the truth values of the following statements. The domain is the set of real numbers. Carefully explain your answer.

a.  $\forall x \forall y (x + y > 0)$       c.  $\exists x \forall y (x + y > 0)$

b.  $\forall x \exists y (x + y > 0)$       d.  $\exists x \exists y (x + y > 0)$

3. [10 pts] We have seen several versions of De Morgan laws. Complete the following:  
(Just state the laws, no justification or proof is necessary)

De Morgan Laws for Logic:

$$\neg(p \wedge q) =$$

$$\neg(p \vee q) =$$

Generalized De Morgan Laws for Logic:

$$\neg(\forall x P(x))$$

$$\neg(\exists x P(x))$$

4. [15 pts] Using a direct proof, show that:

*For all integers  $m$  and  $n$ , if  $m$  and  $n$  are odd, then  $mn$  is odd.*

5. [15 pts] Using a proof by contra-positive, show that:

*For every integer  $m$ , if  $m^2$  is odd, then  $m$  is odd.*

6. [15 pts] Using induction, show that:

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}, \text{ for all } n \geq 1$$

(You need to give the complete proof, including base case etc to get full credit.)

7. [15 pts] Using induction, show that:

$7^n - 1$  is divisible by 6, for all  $n \geq 1$

(You need to give the complete proof, including base case etc to get full credit.)