

Exam 2 Math 142 Calculus

1. [10 pts] A continuous function defined for all x has the following properties:
f is decreasing, f is concave up, $f(5) = -2$ and $f'(5) = -1$
 - a. Sketch a possible graph for f
 - b. How many zeros does f have? Why?
 - c. What can you say about the location of the zeros? Explain.

2. [10pts] On what interval(s) is the function $f(x) = \ln(x^2 + 1)$ concave up? You may use the calculator to check your work, but you should use calculus to find the answer exactly. Show work.
3. [10pts] (a) When is a function differentiable at x ? (i.e. give the definition)
(b) Give a graph (i.e. make one up) of a function that is continuous but not differentiable. Explain where it is not differentiable and why.

4. [10pts] (a) Give the definitions of $\sinh(x)$ and $\cosh(x)$.
(b) Show from definitions that $\frac{d}{dx} \cosh(x) = \sinh(x)$
5. [10pts] Find the quadratic polynomial $g(x) = ax^2 + bx + c$ which best fits the function $f(x) = \sin(x) + \cos(x)$, in the sense that $f(0) = g(0)$, $f'(0) = g'(0)$, and $f''(0) = g''(0)$. (In other words solve for a, b, and c.)
6. [10pts] If the derivative of $y = k(x)$ equals 2 when $x=1$, what is the derivative of:
 - a. $k(3x)$ when $x = \frac{1}{3}$?
 - b. $k(x - 1)$ when $x = 2$?
 - c. $k(\frac{1}{5}x)$ when $x = 5$?

7. [10pts] Show, using the properties of inverse functions, that $\frac{d}{dx} \ln(x) = \frac{1}{x}$. Show work!
8. [10pts] (a) Show that $1 - x$ is the local linearization of $\frac{1}{1+x}$ near $x = 0$.
(b) Using (a), near $x = 0$ what is $\frac{1}{1+x^2}$ approximately equal to? (Hint use substitution).

6. Suppose f is continuous and differentiable on the interval $[1,5]$.
 - A. If $f(1)=2$ and $f(5)=7$, what conclusion can be drawn using the Intermediate Value Theorem?
 - B. If $f(1) = 0$ and $f(5) = 0$, what conclusion can be drawn using the Mean Value Theorem?

1. [10pts] Show, using the properties of inverse functions, that $\frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$.

Carefully show all your work.

3. [10pts] Are there any x -values at which the absolute value function $f(x) = |x^2 - 1|$ is:

- a. Not continuous?
- b. Not differentiable?

Show work and explain!

7. [15pts] Is the graph of $f(x) = \cos(x^4)$ increasing or decreasing when $x=10$? Is it concave up or concave down? Show work and explain!

9. Suppose that $f(x) = \ln(x) + x^{-1}$

- a. What is the derivative of $f(x)$?
- b. Where is the derivative of f equal to zero?
- c. Graph $f(x)$. What happens on the graph of f at the point where the derivative is zero?

If $g(v)$ is the fuel efficiency, in miles per gallon, of a car going at v miles per hour, what are the units of $g'(90)$? What is the practical meaning of the statement $g'(60) = -0.59$?

**These are just examples of exam problems!!
Make sure you go over your homework!**