

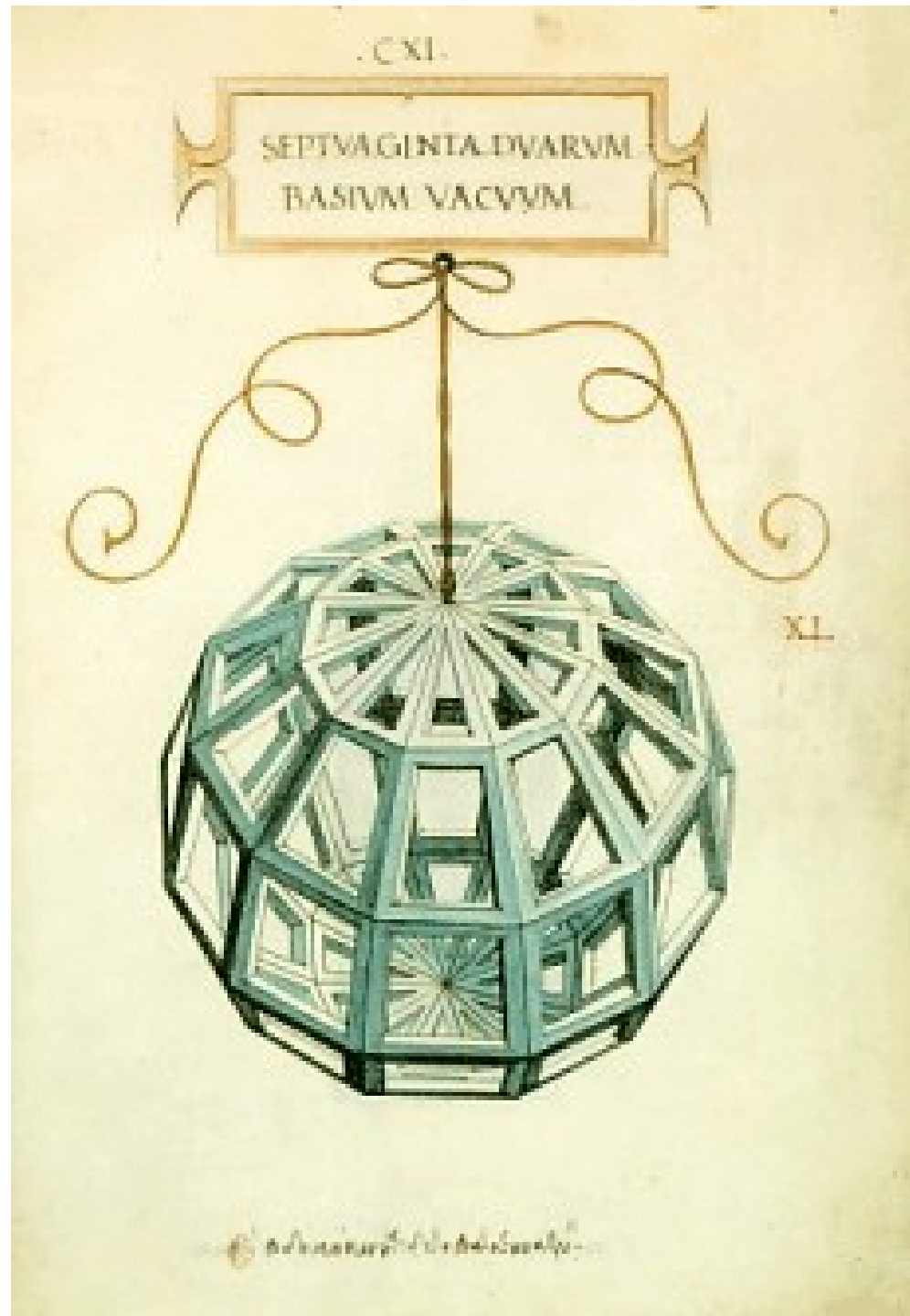
# Geometry in two dimensions

- Euclidean geometry
  - Euclid (3<sup>rd</sup> Century BC)
- Spherical geometry
  - Eratosthenes (3<sup>rd</sup> Century BC)
- Hyperbolic geometry
  - Lobachevsky, Gauss (~1830)

# Neolithic stone spheres (c2000BC)



Leonardo da Vinci  
Vinci  
(*The Divine Proportion*,  
1509)



# Spaceship Earth, 1982



# Outline

- Why study non-Euclidean geometry?
- Spherical Geometry
- Spherical Tessellations
- Polyhedra

# Why Study Non-Euclidean Geometry?

- M.C. Escher (Sphere with Angels and Devils, 1942)



# Why Study Non-Euclidean Geometry?

- Spherical & hyperbolic tessellations.
- Polyhedra and Platonic solids.
- Good source of student artwork ideas.
- Beautiful classification of regular tessellations.
- Possible influence on Cubism (early 1900's).

# Spherical Geometry

- Geometry on the surface of a sphere.
- “Geo-metry”: measuring the Earth
- Geometry requires points and lines. What are the lines?

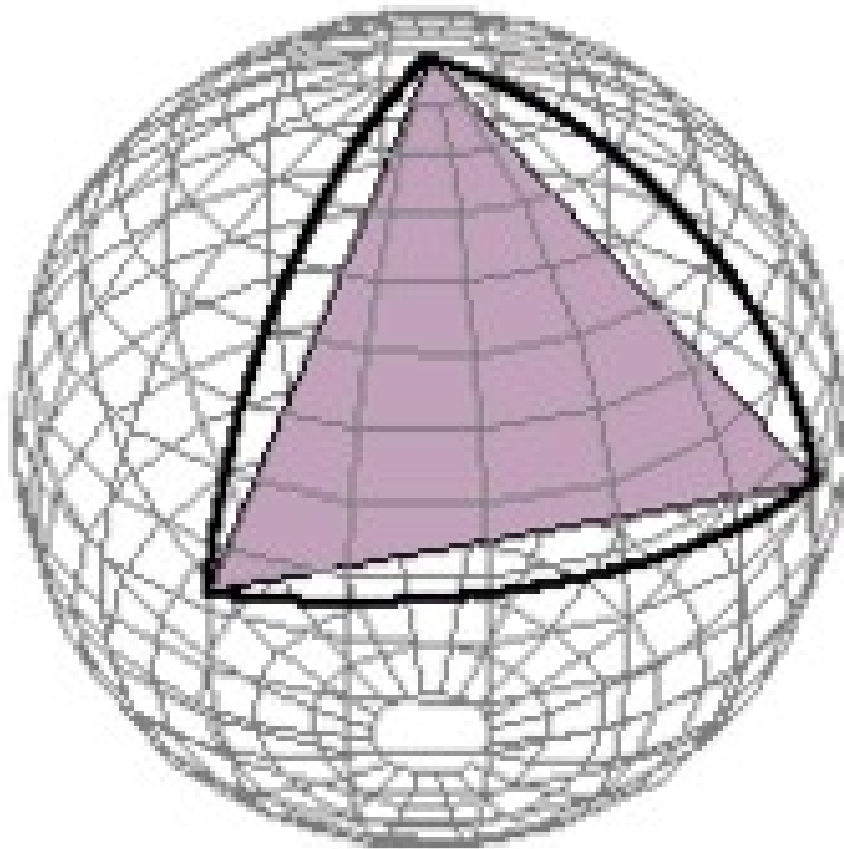
# Geodesics

- Geodesics play the role of “straight lines”
  - Look straight to an observer on the surface of the sphere
  - Shortest distance paths
- Spherical geodesics are great circles.



# Triangles

- Made of three geodesic segments.
- Have angle sum  $> 180^\circ$ . They “bulge”.



# Defect and area

- The defect of a triangle is the amount its angle sum differs from  $180^\circ$ .
- The defect of any polygon is the amount its angle sum differs from the angle sum of a Euclidean polygon with the same number of sides.
- $\text{Defect} / 720^\circ = \text{Fraction of Sphere Covered}$ .

# Modalities

- 2D projected drawing
- Drawing on a sphere
- Using software