

Symmetry, as wide or as narrow as you may define its meaning, is one idea by which man through the ages has tried to comprehend and create order, beauty, and perfection.

- Hermann Weyl

Plan:

- Mathematical Background
 - Review of main concepts.
- Example symmetry worksheets.
- Teaching Symmetry
 - How far to go?
 - Practices and pitfalls.
- Sources of examples.

Symmetry

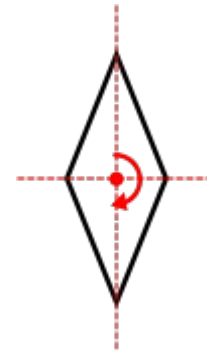
- An object is preserved by a transformation.
- Object: Subset of the Euclidean plane.
- Transformation: Isometry (rigid motion).
- Other examples:
 - Subsets of Euclidean 3-space, n-space
 - Subsets of a sphere, of hyperbolic space
 - Similarity transformations (rigid motion + rescaling)

Isometries of the Plane

- Complete classification into four types:
 - Reflection
 - Rotation
 - Translation
 - Glide Reflection

Symmetry Group

- A symmetry of a plane subset S is an isometry that preserves S .
- The set of symmetries of S forms a group.
- Example: The diamond has four symmetries, two reflections, a 180° rotation, and the trivial symmetry.













Classification

- A symmetry group with no symmetry ϵ -close to the identity is called **discrete**.
- Complete classification of discrete symmetry groups of the plane:
 - Rosette groups (finite)
 - Frieze groups (one translation axis)
 - Wallpaper groups (lattice of translations)

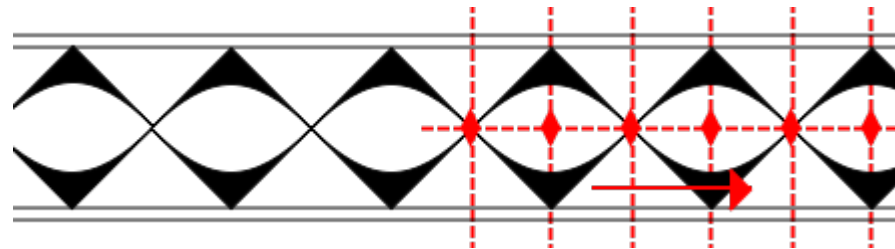
Rosette Groups

- The only finite symmetry groups in the plane.
- Cyclic groups: C_1 (trivial), C_2 , C_3 , C_4 ,...
- Dihedral groups D_1 (bilateral), D_2 , D_3 , D_4 ,...

Rot. order	1	2	3	4	5
Cyclic					
Dihedral					

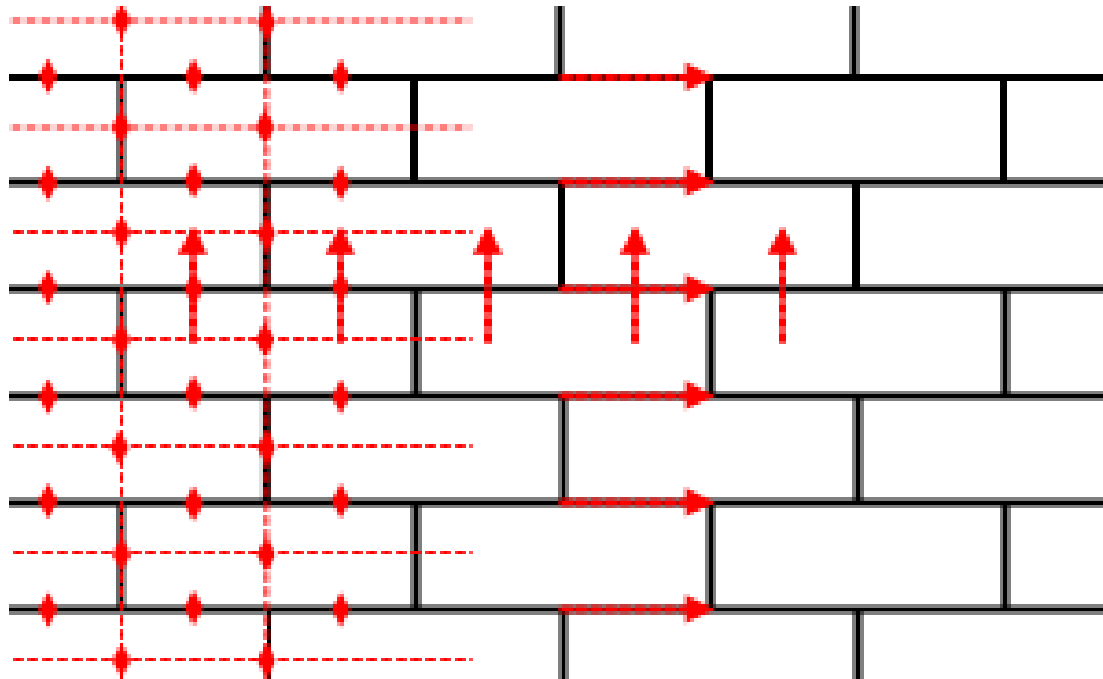
Frieze Groups

- Also known as border groups, strip groups, or one-dimensional groups.
- Seven of these.
- Example: San Ildefonso pottery decoration, with symmetry group $pmm2$.



Wallpaper Groups

- 17 of these.
- Example. This brick pattern has translations, rotations, reflections, and glide reflections. It has symmetry group *cmm*.



Color Symmetry

- Considering a pattern as a subset of the plane is limiting.
- Can extend the classification of discrete symmetry groups to multicolored patterns.

