

Exam Questions

- Draw a sphere with
 - some great circles on it.
 - a 90° - 90° - 90° triangle on it.
 - three geodesics that meet at right angles.
 - a biangle
- Where is the image under 90° rotation of this fish?
- Does Angle-angle-angle hold in spherical geometry? Explain.

Counting Faces of Polyhedra

- Use symmetry to infer the corner angles.
- Calculate the defect.
- Calculate the area fraction, and take the reciprocal.

Concentric Rinds

- 45-90-60 triangles.
- Angle sum 215, defect is 35
- Area fraction $35/720 = 1/48$
- 48 faces
- (Or use symmetry!)

Regular tessellations of the sphere

- Triangles.. three angles x .
- $x+x+x > 180^\circ$, so $x > 60^\circ$
- Must fit around a vertex (x divides 360°)
- Possibilities:
 - 3 at a vertex, $x=120^\circ$, each has area $1/4$, use 4.
 - 4 at a vertex, $x=90^\circ$, each has area $1/8$, use 8.
 - 5 at a vertex, $x=72^\circ$, each has area $1/20$, use 20.
 - 6 at a vertex, $x=60^\circ$, flat! More than 6, $x < 60^\circ$.

Platonic Solids

- Complete classification of regular tessellations.
- “Flat versions” are the Platonic Solids
 - tetrahedron
 - octahedron
 - icosahedron
 - cube
 - dodecahedron

Plato – *The Timaeus*

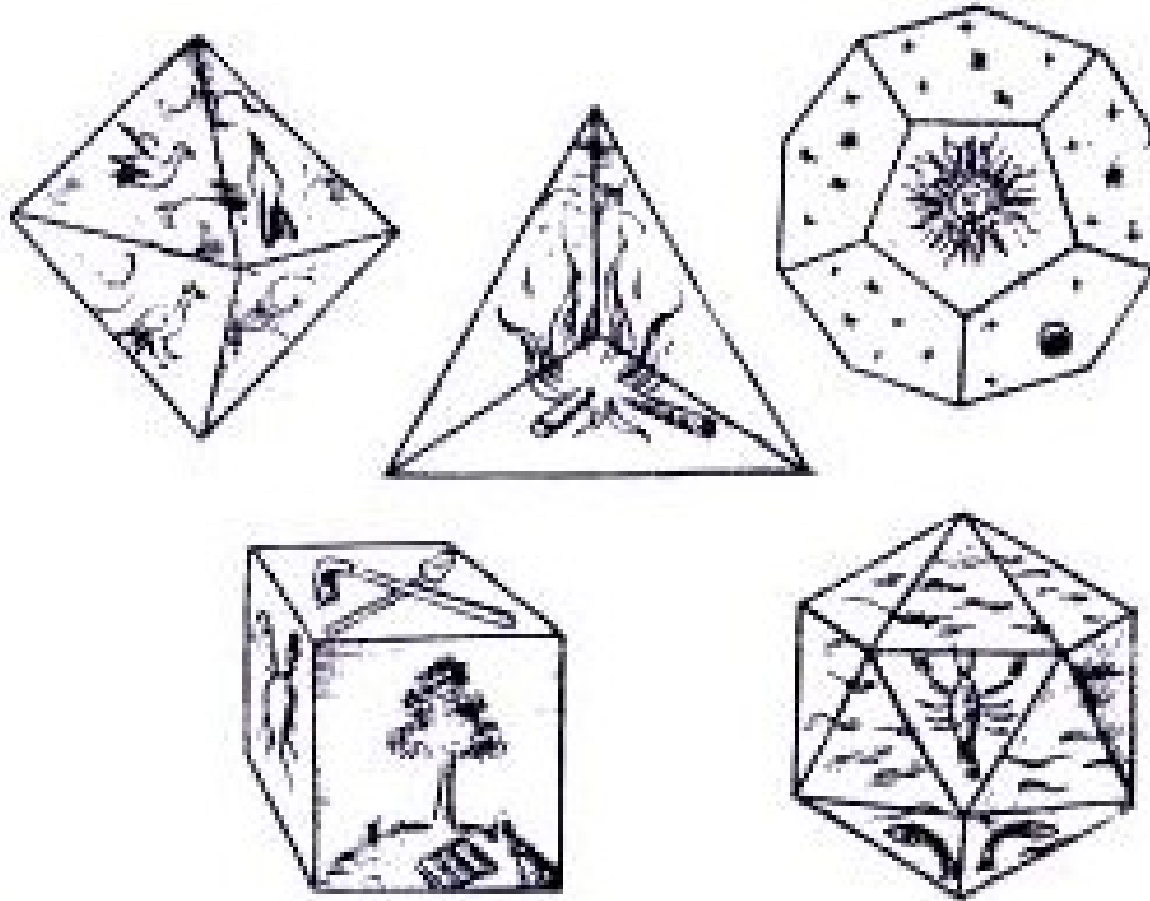
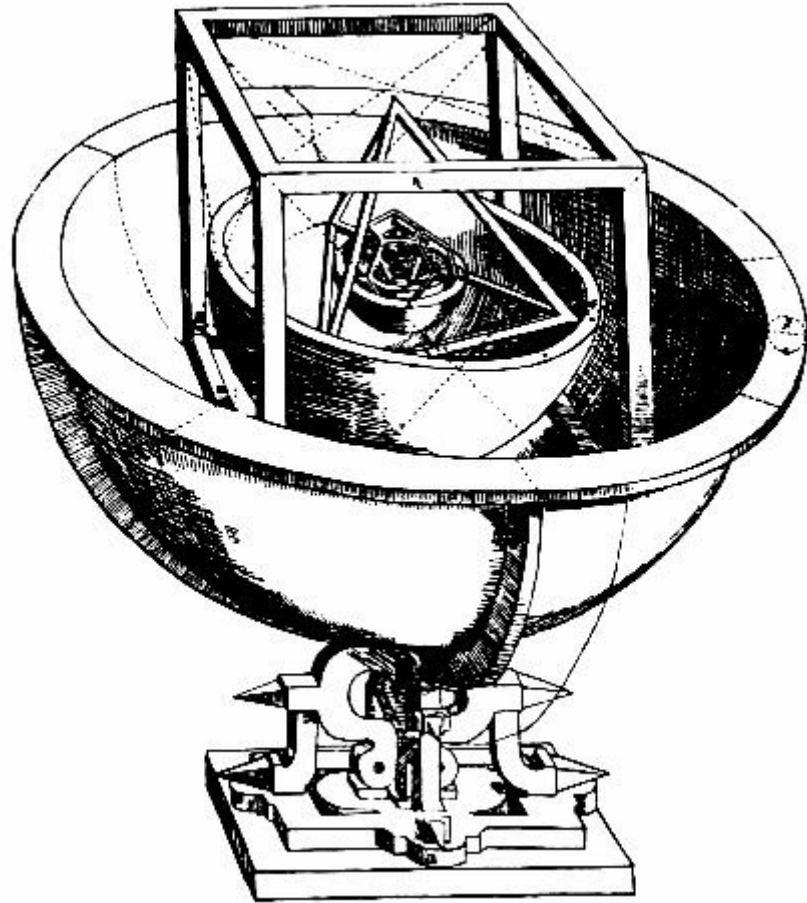


Illustration by Johannes Kepler, *Harmonice Mundi*, 1619

Johannes Kepler (1571-1630)

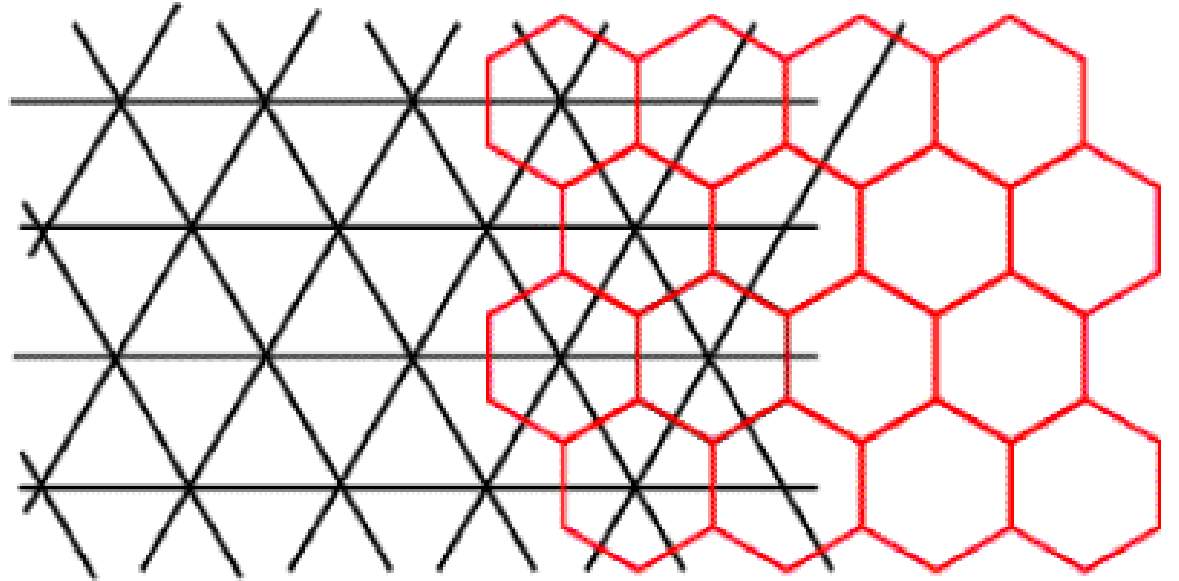


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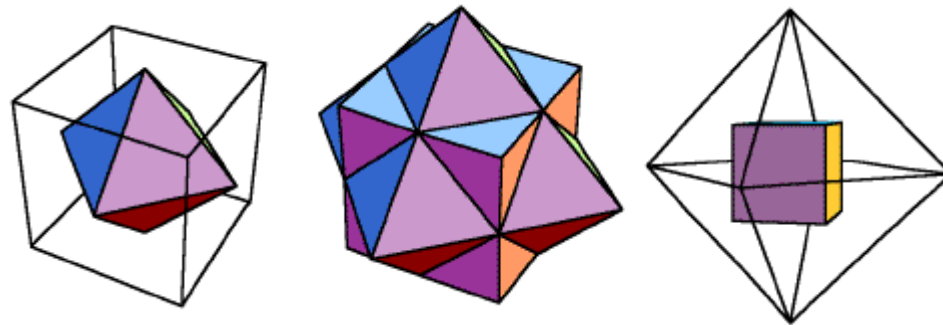
-However, there are, as it were, two noteworthy weddings of these figures, made from different classes: the males, the cubes and the dodecahedron, among the primary; the females, the octahedron and the icosahedron, among the secondary, to which is added one, as it were, bachelor or hermaphrodite, the tetrahedron, because it is inscribed in itself, just as those female solids are inscribed in the males and are, as it were, subject to them, and have the signs of the feminine sex, opposite the masculine, namely, angles opposite planes.

Duality

- Dual tessellations



- Dual Polyhedra



Combinatorics of Polyhedra

- Take counting farther and have students count vertices, edges, faces.
- Euler characteristic: $V - E + F = 2$.
- $\# \text{ Edges} = (\text{Edges per face}) * (\# \text{ Faces}) / 2$

Pentakis Dodecahedron

- Triangles are 60° - 60° - 72°
- Defect 12° , area fraction $12/720=1/60$
- There are 60 faces
- $60 \text{ faces} * 3 \text{ edges per face} / 2 \text{ times counted} = 90 \text{ edges}$
- $60 - 90 + V = 2$, so $V = 32$

Sources

- George Hart, Polyhedra and Art
 - <http://www.georgehart.com/>
- Paul Calter, Geometry in Art and Architecture
 - <http://www.math.dartmouth.edu/~matc/math5.geometry>