Triangles , Math 181 , Informal Geometry

Given a triangle ABC, we can construct four different types of lines with respect to the triangle.

1. The **angle bisector** bisects an angle to form two congruent angles.
2. The **perpendicular bisector**. Given a line segment, the perpendicular bisector is the unique perpendicular line passing through the midpoint of the line segment.
3. The **median** is the line passing through a vertex and the midpoint of the opposite side.
4. The **altitude** is the line passing through a vertex, perpendicular to the opposite side.

Properties of the angle bisector:

- Any point on the angle bisector is equidistant from the sides which form the angle.
- The three angle bisectors in a triangle always intersect in one point, and this intersection point always lies in the interior of the triangle.
- The intersection of the three angle bisectors forms the center of the circle inscribed in the triangle. (The circle which is tangent to all three sides.)

Properties of the perpendicular bisector:

- Any point on the perpendicular bisector of a line segment is equidistant from both endpoints.
- In a triangle the perpendicular bisectors of the three sides always meet in a single point. This point is called the circumcenter.
- If the triangle is acute, the circumcenter lies inside the triangle. If the triangle is obtuse, the circumcenter lies outside the triangle. If the triangle is a right triangle, the circumcenter will coincide with one of the sides.

- The circumcenter is the center of the circumscribed circle. (The circle which passes through all three vertices.)

Properties of the median:

- The medians of a triangle always intersect in one point (the centroid).
- The centroid always lies inside the triangle.
- The centroid divides the median into two segments. The lengths of these two segments always have a constant ratio.

Properties of the altitude:

- The altitudes of a triangle always intersect in one point.
- If the triangle is acute, the intersection point lies inside the triangle. If the triangle is obtuse, the intersection point lies outside the triangle. If the triangle is a right triangle, the intersection point will coincide with the vertex which represents the right angle.

Some more Geometer's Sketchpad exercises.

1. The median is divided into two pieces whose length have a constant ratio.
   Start a new sketch. Draw a triangle. Select all three of the sides and construct the three midpoints. Now draw the each of the three medians by selecting a vertex and the midpoint of the opposite side and choosing ''construct line".
   Once you have the three medians, select two of the medians and draw their intersection point by choosing ''construct intersection point".
   Pick one of the medians and find the length of the two line segments. To do this, first select the vertex and the intersection point and select ''measure distance". Then select the intersection point and the midpoint of the opposite side and select ''measure distance".
   Go to the measure menu and select ''calculate..". We can now calculate the ratio of the lengths of the line segments by simply selecting one of the measurements, clicking on the division sign on the calculator pad and then selecting the other measurement. Hitting the return key will display the ratio on your GSP window. What is this ratio?
   Select one of the vertices of the triangle and check that this ratio does not change if you change the triangle.

2. When do the angle bisectors and the perpendicular bisectors coincide?
   This question can be asked in two slightly different ways.

   (a) Start a new sketch. Draw an acute triangle. Label the vertices. Select A,B and C (in that order) and choose ''construct the angle bisector". This gives us the angle bisector for angle B. Select this angle bisector and use ''display color" to change the color of the angle bisector from black to a color of your choice.
   Now select line segment AC and construct the midpoint. (You need to go back to ''display color" and change the color to something other than the color you chose for the angle bisector.) Now select AC and the midpoint and select ''construct a perpendicular line".
   Select point C and move it about the screen until the angle bisector and the perpendicular bisector coincide. Can you move vertex B in any way so that the angle bisector and the perpendicular bisector still agree? What properties does the triangle have? (Be as general as possible.)

   (b) Now start a new sketch. Draw an acute triangle. Construct all three angle bisectors. Select all three angle bisectors and using ''display color" change their color. Next click somewhere in the window so that nothing

is selected. Go to ''display color" and change to another color. Now construct all three perpendicular bisectors. Manipulate the three vertices until the angle bisectors and the perpendicular bisectors all overlap. What kind of triangle do you have? How do you know? (I.e. how did you check this? How many different ways can you think of to check this?)

3. When do medians and the perpendicular bisectors coincide?
This question can be asked in two slightly different ways.

   (a) Start a new sketch. Draw an acute triangle. Label the vertices. Select line segment AC and construct the midpoint. Draw the median by selecting the midpoint and vertex C and choosing ''draw line". Select the median. Go to ''display color" and change the color of the median.
   Now make sure no part of your figure is selected. You need to go back to ''display color" and change the color to something other than the color you chose for the median. Now select the midpoint of AC and select ''construct a perpendicular line".
   Select point C and move it about the screen until the median and the perpendicular bisector coincide. Can you move vertex B in any way so that the median and the perpendicular bisector still agree? What properties does the triangle have? (Be as general as possible.)

   (b) Now start a new sketch. Draw an acute triangle. Construct all three medians. Select all three medians and using ''display color" change their color. Next click somewhere in the window so that nothing is selected. Go to ''display color" and change to another color. Now construct all three perpendicular bisectors. Manipulate the three vertices until the medians and the perpendicular bisectors all overlap. What kind of triangle do you have? How do you know? (I.e. how did you check this? How many different ways can you think of to check this?)