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# Informal Geometry

Math 181 Spring 2000

Tessellations

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We have looked at tessellations in a couple of different ways now.

We will now look at tessellations of the plane by regular polyhedra.

What is the definition of a regular polygon?

What is the common name for a regular 3-gon?

What is the common name for a regular 4-gon?

A tessellation is a **regular tessellation** if it is constructed from regular convex polygons of one size and one shape.

There are exactly three regular polygons that tessellate the plane. Below are the tessellations of the plane by an equilateral triangle, a square, and a regular hexagon.

The book defines what is called a **vertex figure**. The vertices are the midpoints of the polygonal edges emanating from a vertex of the tessellation. For each of the three regular tessellations, draw the vertex figures.

Are they the same at every vertex?

What type(s) of vertex figures do you get for the regular tessellations?

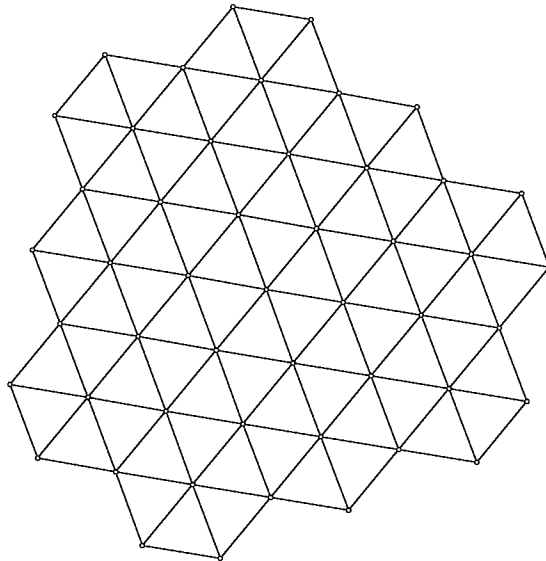


Figure 1: Regular Tessellation by Equilateral Triangles

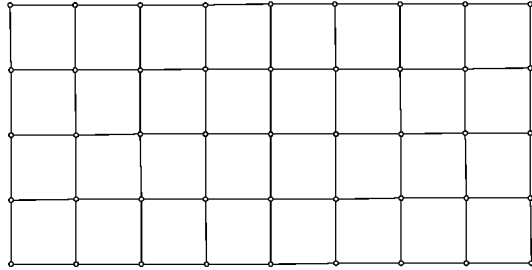


Figure 2: Regular Tessellation by Squares

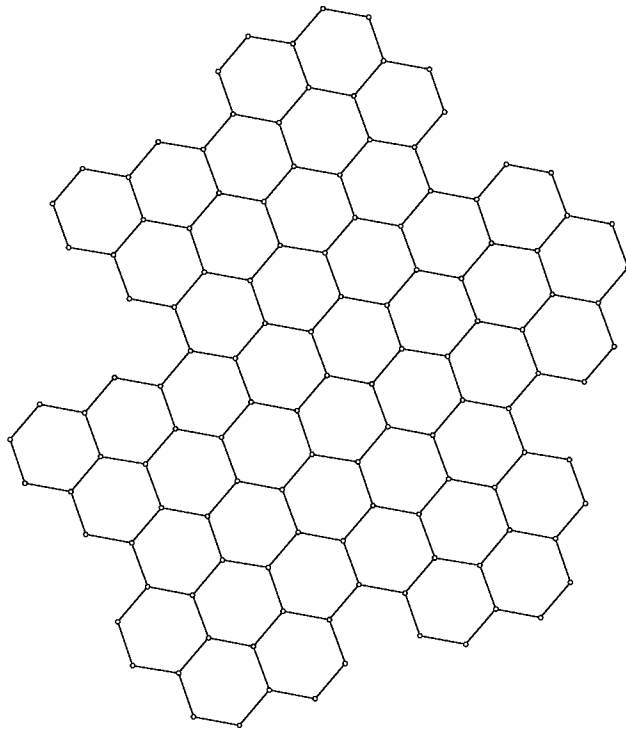


Figure 3: Regular Tessellation by Regular Hexagons

The regular pentagon does not tessellate the plane. The figure below shows that when we try to arrange the pentagons around a vertex, then we will always have a gap or an overlap.

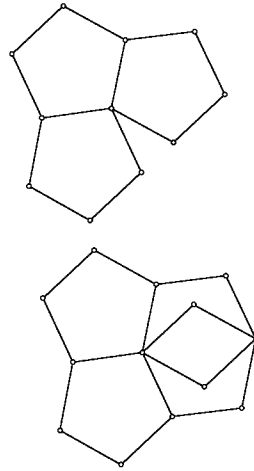


Figure 4: 3 pentagons leave a gap, but 4 pentagons will overlap

Note that there are non-regular pentagons which tessellate the plane. Sketch the tessellation for a pentagon which looks like the outline of a house to illustrate this point.

We can also look at tessellations formed by a combination of several regular polygons. We say that a tessellation is a **semi-regular tessellation** if it is composed of regular polygons of two or more types so that all the vertex figures are the same. For the three tessellations below (fig. 5,6 and 7) extend the tessellations. Then draw the vertex figures and decide if the tessellation is regular. Explain why it is or is not semi-regular.

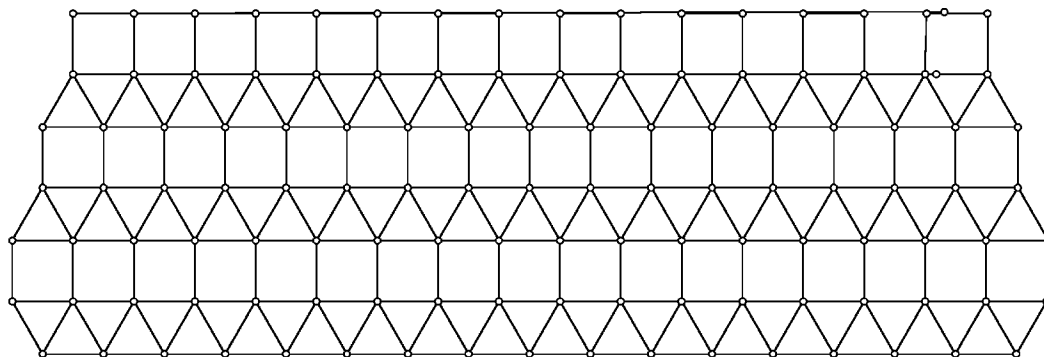


Figure 5: Is this a semi-regular tessellation?

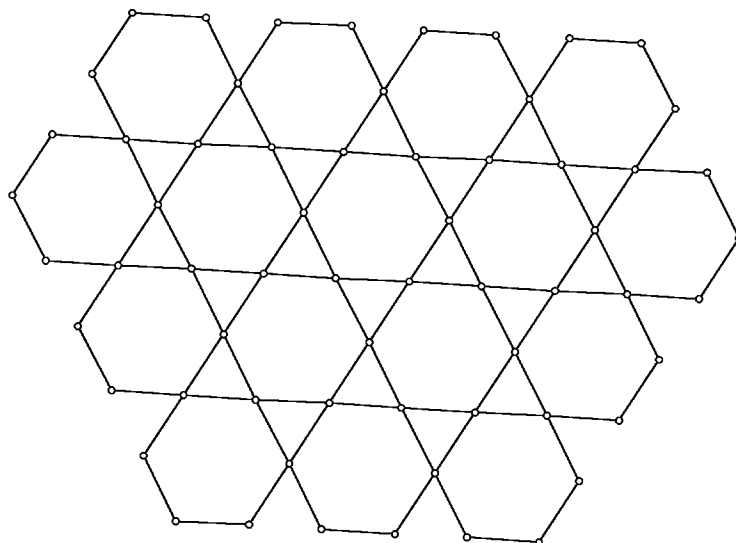


Figure 6: Is this a semi-regular tessellation?

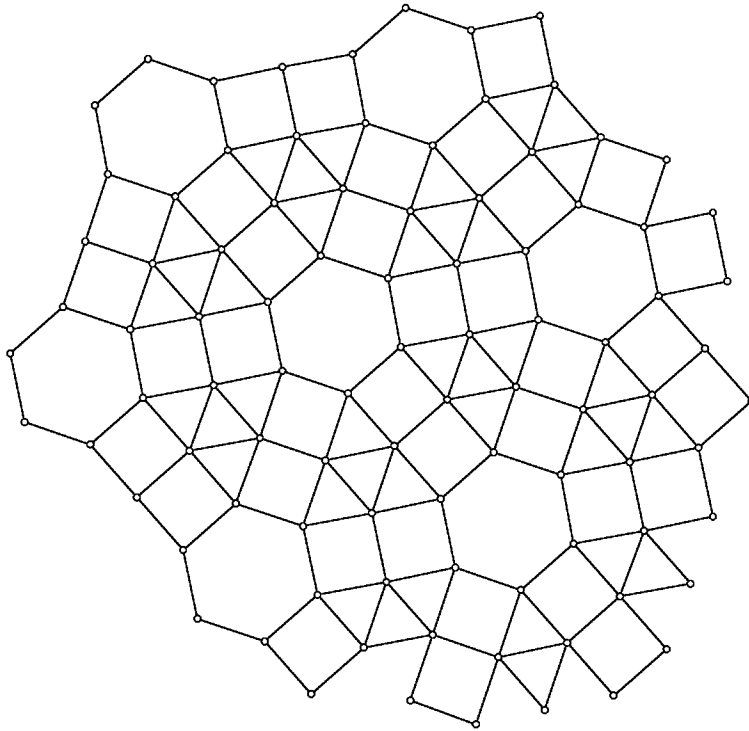


Figure 7: Is this a semi-regular tessellation?

The book shows that there are only 8 possible semi-regular tessellations! (See page 105 and 106).

Copy the eight semiregular tessellations, and write down the vertex types. For instance, the top left tessellation on page 105 is of type  $(8,8,4)$ , because at every vertex we see (in order) a 8-gon, another 8-gon and a 4-gon (in regular english that would be octagon,octagon,square).

(Remark:  $(8,8,4)$  is the same as  $(8,4,8)$  and  $(4,8,8)$ . It depends on where you start to count!)