Animating Taylor polynomials

MapleNet - Demonstration

Demonstration Worksheet by Mike May, S.J.- maymk@slu.edu

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This worksheet is an adaptation of an older worksheet on animating Taylor series. It is intended as a demonstration of what can be done with MapleNet. For that reason it includes more text on the construction of the worksheet.

First some technical details. We restart Maple to clear memory and load some packages we need. The button below executes the commands:
restart:
with(plots):
with(Student[MultivariateCalculus]):
with(DocumentTools):

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Restart and load packages

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When working with Taylor polynomials there are two features that are worth demonstrating with animations that compare higher and higher degree Taylor polynomials with the original function. The first feature is that, as we would hope with something that is called an approximation, the Taylor polynomials morph into the function. The second feature is the radius of convergence. If the Taylor series has a finite radius of convergence, it will only morph to the function in that region, and will never become a good approximation outside that region.

The animations are memory intensive. Since this often causes memory problems the animation will be done as a separate worksheet. Before doing this worksheet, you should have worked through the worksheet on Taylor polynomials in several variables.

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One variable convergence demonstration

We start with initial information about the functions and where we want to look at the Taylor polynomial.

1/(1+x^2)
function(x) = x0 = 1 , width from center = 2.5 ,

height from center = 2.75

mi deg = 2 , num steps = 10 , deg/ step = 2

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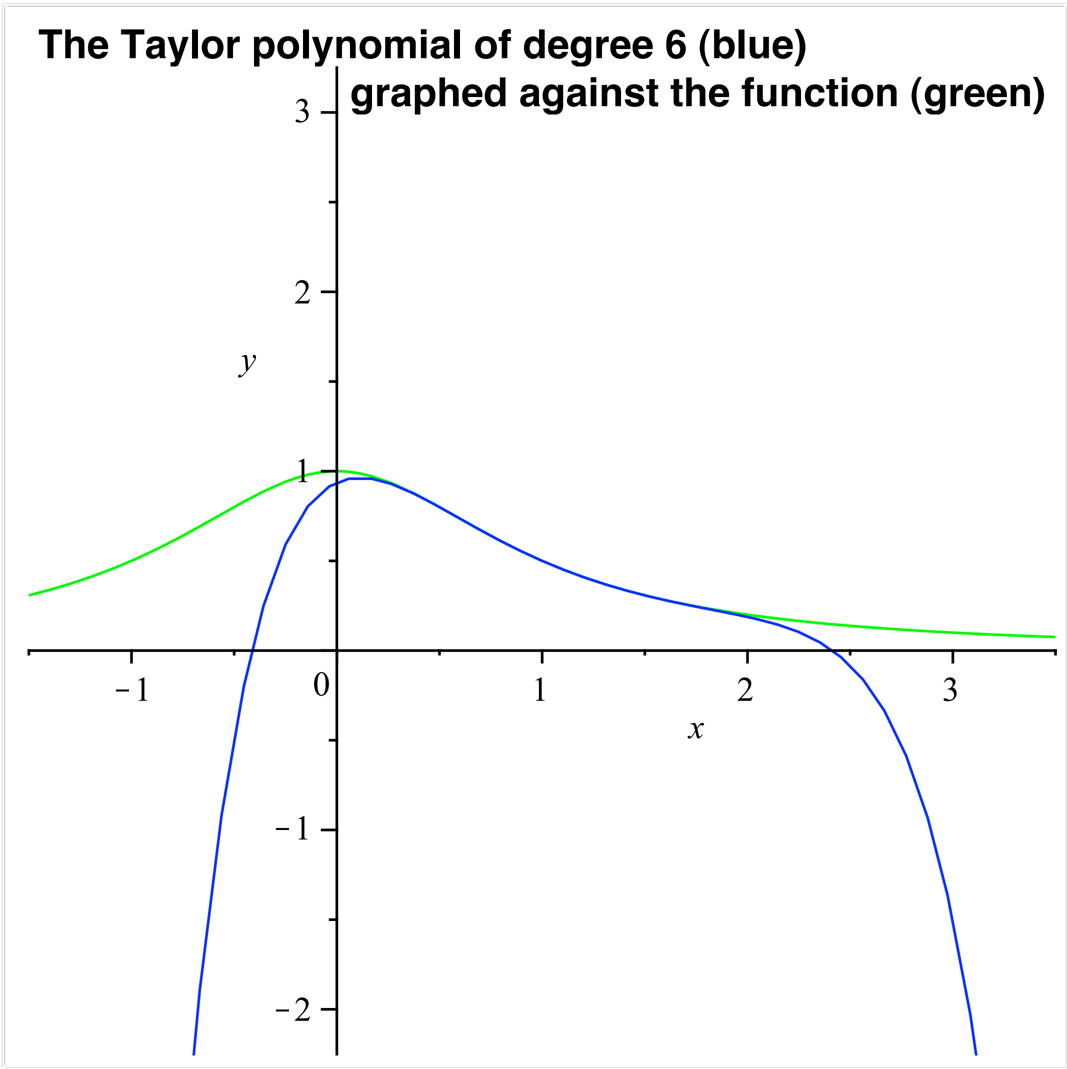
Plot Taylor Animation



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Desired Taylor polynomial degree = 10

Compute Taylor Polynomial

$$\frac{1}{362880}x^9 - \frac{1}{5040}x^7 + \frac{1}{120}x^5 - \frac{1}{6}x^3 + x$$

▶ **Construction Details**

One variable radius of convergence demonstration

Next we want to demonstrate the idea of radius of convergence.
We want to look at what happens if the series has a finite radius of convergence.

For the finite radius of convergence we change:
the function to 1/(1+x^2)
x0 to 1.5
width to 3.5

On a second pass through the series of graphs we can raise the degrees per step and the number of steps to 20 so that we are looking at up to a 400 degree polynomial.

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