

# A Fast Guide to Maple and Partial Derivatives

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```
[> restart;
```

## ▼ Preliminaries - Establishing Functions

To use Maple to find partial derivatives we first need to be able to define functions. We start with the simple function  $f$  taking  $(x,y)$  to  $x^2 + y^3$ . This can be done in a number of ways.

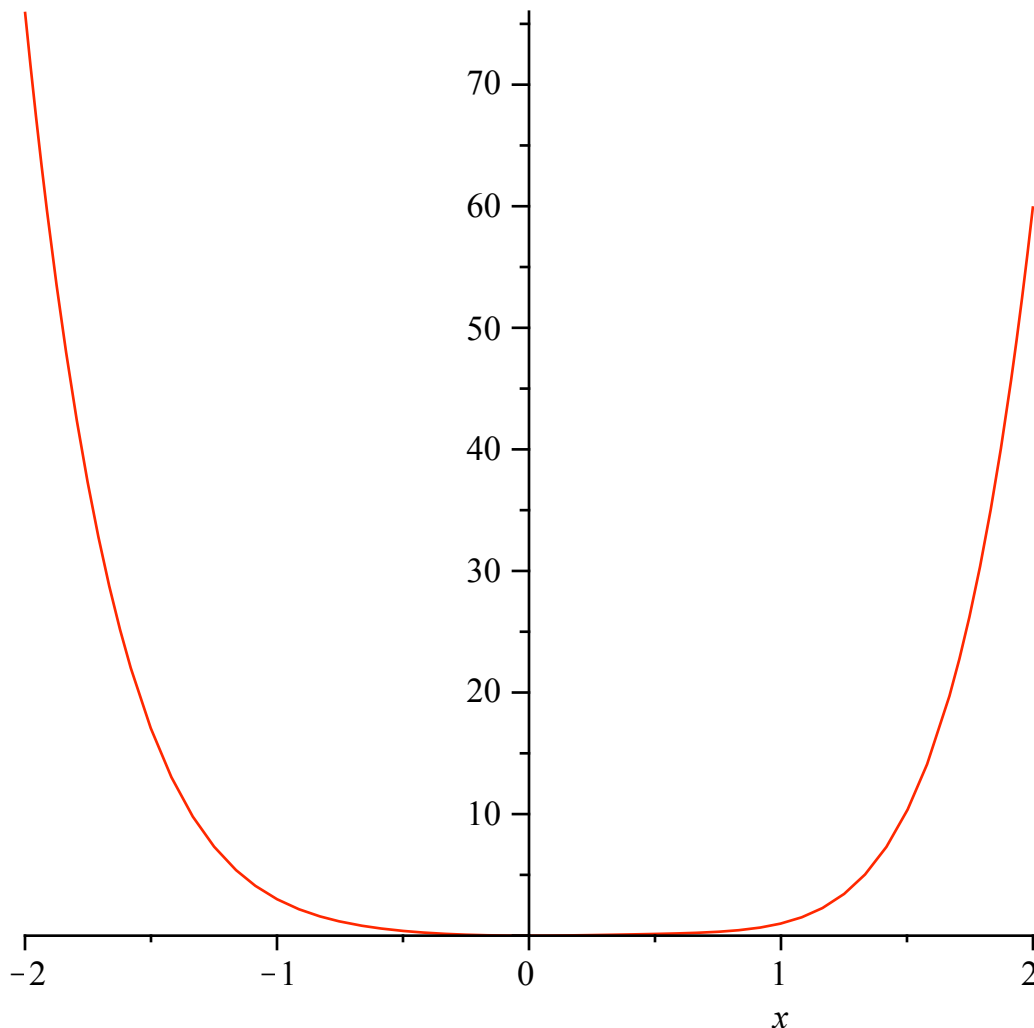
1) Use the insert menu to insert an execution group after the cursor (command-J), insert Maple input (command-M) getting a vertical cursor and red type, and define the function with the syntax "f := (x,y) -> x^2+y^3;" We then evaluate the function for a particular  $(x,y)$ .

```
> f := (x,y) -> x^2+y^3-x*y;
```

$$f := (x, y) \rightarrow x^2 + y^3 - xy$$

(1.1)

```
> plot(f(x, x^2), x=-2..2);
```



```
> h := x^2+y^3-x*y;
```

$$h := x^2 + y^3 - xy$$

(1.2)

```
> eval(h, x=2);
subs(x=2, h);
```

$$4 + y^3 - 2y$$

$$4 + y^3 - 2y$$

(1.3)

```
> h(1, 2);
```

$$x(1, 2)^2 + y(1, 2)^3 - x(1, 2)y(1, 2)$$

(1.4)

```
> f(2, z);
```

$$4 + z^3 - 2z$$

(1.5)

2) Use the same method in 2-D math mode. (On a new line insert 2-D Math, command-R.) Note that Maple turns the dash-greater than combination into an arrow.

$g := (x, y) \rightarrow x^2 + y^3;$

$$(x, y) \rightarrow x^2 + y^3$$

(1.6)

```
> g(t, t);
```

$$t^2 + t^3$$

(1.7)

```
>
```

3) Use the 2-D math mode and the Expression palette to the left. Click on the expression  $f:=(a,b) \rightarrow z$ , then navigate through the entries with tabs to fill in the expression.

$f(x, y) \rightarrow x^2 + y^3;$

Error, invalid parameters for inline function

$$f(x, y) \rightarrow x^2 + y^3$$

$h := (x, y) \rightarrow x^2 + y^3;$

$$(x, y) \rightarrow x^2 + y^3$$

(1.8)

$h(k, m);$

$$k^2 + m^3$$

(1.9)

For the exercises, instead of defining a function named  $f$ , you will define a function named  $\text{func}$ , and you will use  $\text{func}$  throughout the exercises.

**Exercise:**

1) Using each of the three methods described above, define the function  $\text{func}(x,y)=\sin(x^2+y^3)$  and evaluate at  $(x,y)=(\sqrt{\pi}/4,0)$ .

```
>
```

## Producing Slice Curves of functions

When we did cross sections of a graph before, we connected a function of two variables with a family of functions in one variables. This can be done by creating new functions. It can also be done by using a constant to fill in one of the variables.

```
> f := (x, y) -> x^2 + y^3 - x*y;
```

```
x0 := 1;
```

```
y0 := 2;
```

```
f1 := x -> f(x, y0);
```

```
f2 := y -> f(x0, y);
```

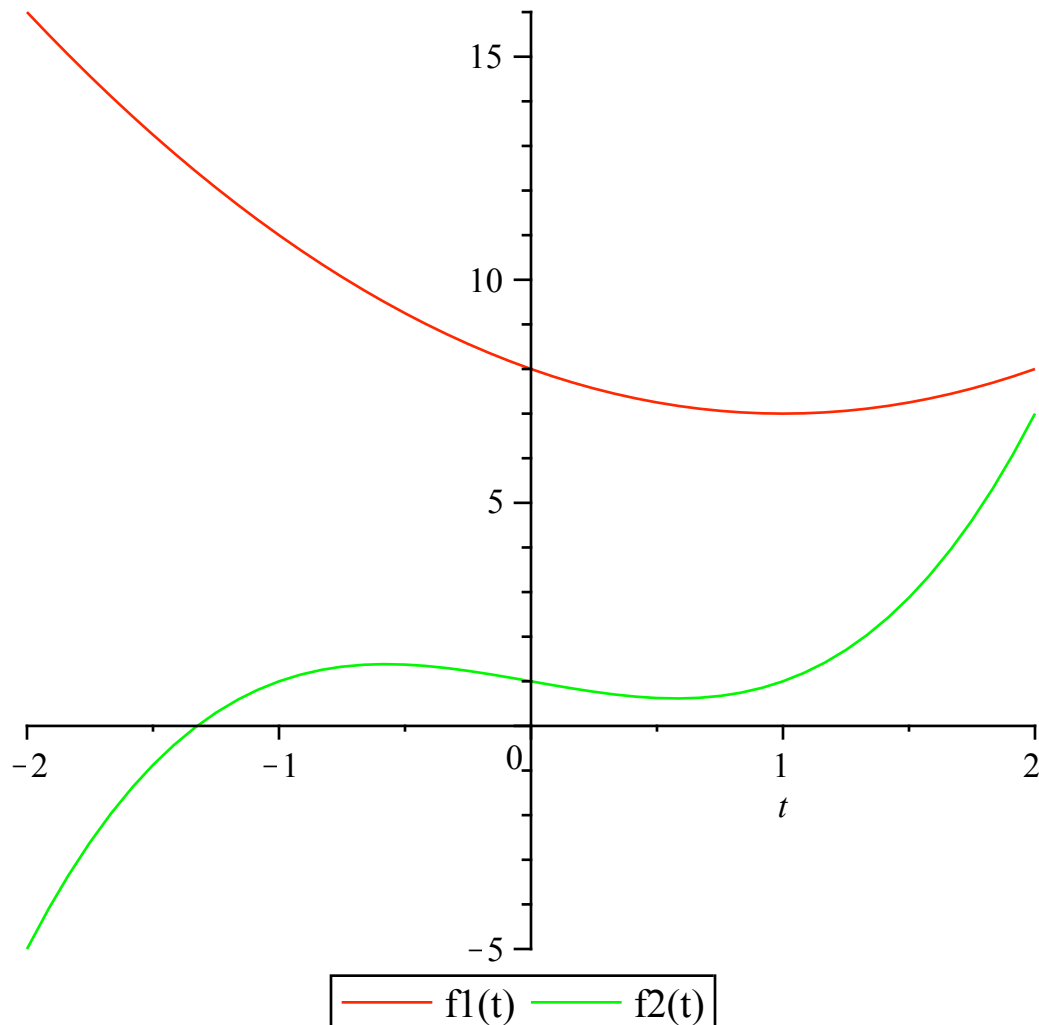
```
plot([f1(t), f2(t)], t=-2..2, color=[red, green], legend =
```

```
["f1(t)", "f2(t)");
```

$$f := (x, y) \rightarrow x^2 + y^3 - xy$$

$$f1 := x \rightarrow f(x, y0)$$

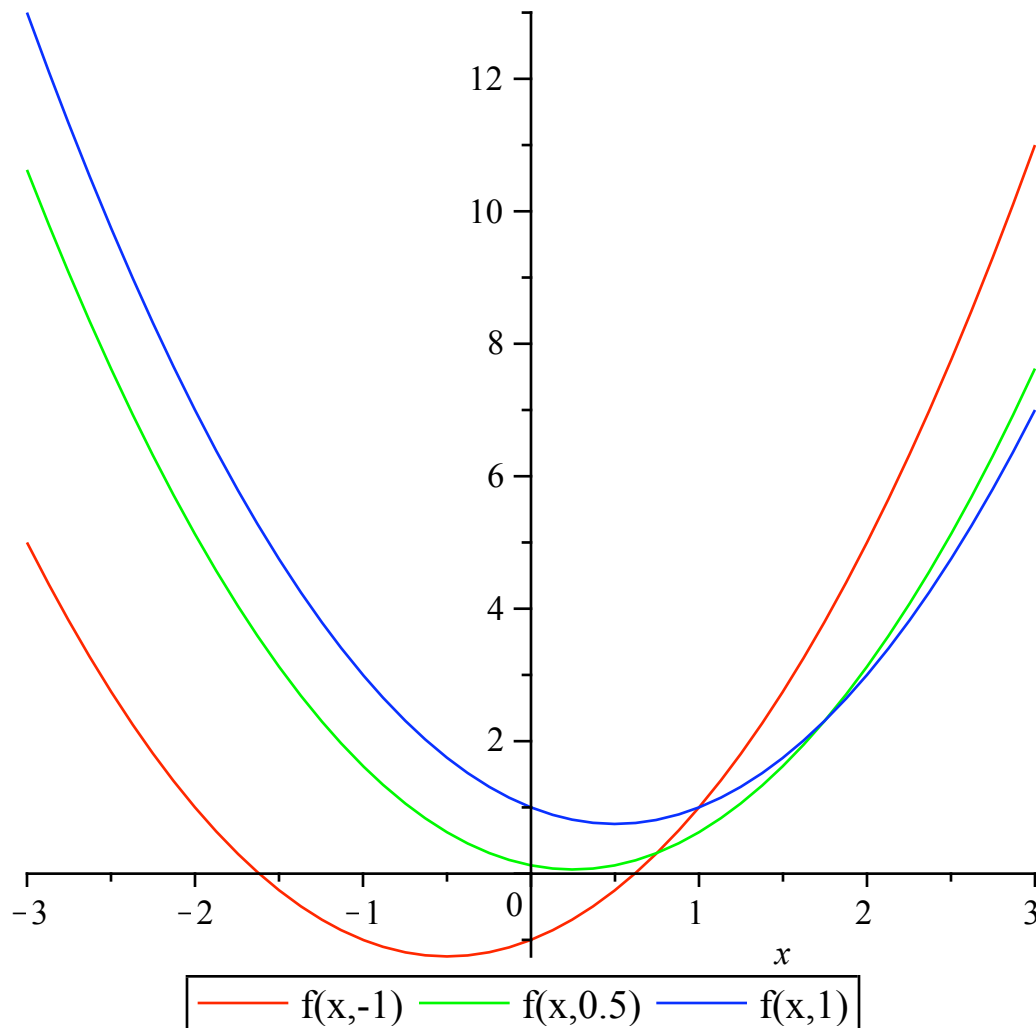
$$f2 := y \rightarrow f(x0, y)$$



We can also plot a family of slices

```
> f := (x, y) -> x^2 + y^3 - x*y;  
plot([f(x, -1), f(x, 0.5), f(x, 1)], x = -3..3,  
color = [red, green, blue], legend = ["f(x, -1)", "f(x, 0.5)", "f(x, 1)"]);
```

$$f := (x, y) \rightarrow x^2 + y^3 - xy$$



**Exercise:**

2) Plot the slice of the function  $\text{func}(x,y)$ , which you defined in exercise 1 above, with  $y=2$ .

## Computing Partial Derivatives

The partial derivative is computed by taking the derivative of a function of two variables, treating one of the variables as a constant.

In Maple, this is done with the `diff` command. It should be noted that the `diff` command has an "inert" version, `Diff`. Applying both commands we see that while Maple uses the same command for the derivative and the partial derivative, it uses a different symbol for the partial derivative.

```
> f := (x,y) -> x^2+y^3-x*y;
f(x,y);
Dfx := diff(f(x,y),x);
Dflx := diff(f(x,y0),x);
Dflxa := diff(fl(x),x);
Diff(f(x,y),x);
Diff(fl(x),x);
Diff(f(x,y0),x);
```

$$f := (x,y) \rightarrow x^2 + y^3 - xy$$

$$x^2 + y^3 - xy$$

$$\begin{aligned}
Dfx &:= 2x - y \\
Dflx &:= 2x - 2 \\
Dflxa &:= 2x - 2 \\
\frac{\partial}{\partial x} (x^2 + y^3 - xy) \\
\frac{d}{dx} (x^2 + 8 - 2x) \\
\frac{d}{dx} (x^2 + 8 - 2x)
\end{aligned} \tag{3.1}$$

$$\begin{aligned}
> \text{Diff}(f(x,y), x) = \text{diff}(f(x,y), x); \\
\frac{\partial}{\partial x} (x^2 + y^3 - xy) = 2x - y
\end{aligned} \tag{3.2}$$

We can do the same computations in 2D math mode or by using the Expression palette.

$$\begin{aligned}
Dfy &:= \frac{\partial}{\partial y} f(x, y); \\
& 3y^2 - x
\end{aligned} \tag{3.3}$$

$$\begin{aligned}
& \frac{\partial}{\partial y} f(x, y); \\
& \frac{\partial}{\partial y} f(x0, y); \\
& \frac{\partial}{\partial y} f(x, y);
\end{aligned}$$

**Exercise:**

3) Compute the partial derivatives of the function func(x,y), which you defined in exercise 1 above, both with respect to x and with respect to y.

$$\begin{aligned}
> 2=3; \\
2 = 3
\end{aligned} \tag{3.4}$$

Finally, we would like to evaluate an expression at specific values. We use the eval command for this. The syntax is

eval(thing to be evaluated, set of values to be used);

This can be done either by typing or using the palette. To get a decimal representation we use the evalf command.

$$\begin{aligned}
> Dfy; \\
3y^2 - x
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
Dfy \Big|_{y=2} \\
12 - x
\end{aligned} \tag{3.6}$$

$$\begin{aligned}
Dfy; \\
3y^2 - x
\end{aligned} \tag{3.7}$$

$$Df_y \Big|_{x=2} \qquad 3y^2 - 2 \qquad (3.8)$$

$$\left( Df_y \Big|_{x=2} \right) \Big|_{y=3} \qquad 25 \qquad (3.9)$$

$$\sin\left(\frac{1}{3}\pi\right); \qquad \frac{1}{2}\sqrt{3}$$

$$\text{evalf}\left(\sin\left(\frac{1}{3}\pi\right), 500\right);$$

0.866025403784438646763723170752936183471402626905190314027903489725966508\ (3.11)

4544000185405730933786242878378130707077033515149849725474994762394058\  
2775604718682426404661595115279103398741005054233746163250765617163345\  
1661443325336127334460918985613523565830183930794009524993268689929694\  
7338251737532880253783091740648030504738010935951625415729147619799164\  
9889491225414435723191645867361208199229392769883397903190917683305542\  
1586890447189158051044152762450835011760355572144347995478182898543584\  
24903645

**Exercise:**

4) Evaluate both func and its partial derivative with respect to y at (2,3).

[>