A Fast Guide to Maple and Partial Derivatives

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```maple
restart;

Preliminaries - Establishing Functions

To use Maple to find partial derivatives we first need to be able to define functions. We start with the simple function $f$ taking $(x,y)$ to $x^2 + y^3$. This can be done in a number of ways.

1) Use the insert menu to insert an execution group after the cursor (command-J), insert Maple input (command-M) getting a vertical cursor and red type, and define the function with the syntax "$f := (x,y) \rightarrow x^2+y^3;$" We then evaluate the function for a particular $(x,y)$.

```maple
> f := (x,y) -> x^2+y^3-x*y;

$$f := (x,y) \rightarrow x^2 + y^3 - x y$$ (1.1)

```maple
> plot(f(x,x^2),x=-2..2);
```

```maple
> h := x^2+y^3-x*y;

$$h := x^2 + y^3 - x y$$ (1.2)

```maple
> plot(f(x,x^2),x=-2..2);
```
eval(h,x=2);
subs(x=2,h);

4 + y^3 - 2 y
4 + y^3 - 2 y

(1.3)

> h(1,2);

x(1, 2)^2 + y(1, 2)^3 - x(1, 2) y(1, 2)

(1.4)

> f(2,z);

4 + z^3 - 2 z

(1.5)

2) Use the same method in 2-D math mode. (On a new line insert 2-D Math, command-R.) Note that Maple turns the dash-greater than combination into an arrow.

g := (x, y) \rightarrow x^2 + y^3;

(x, y) \rightarrow x^2 + y^3

(1.6)

> g(t,t);

f^2 + f^3

(1.7)

> h := (x, y) \rightarrow x^2 + y^3;

(x, y) \rightarrow x^2 + y^3

(1.8)

For the exercises, instead of defining a function named f, you will define a function named func, and you will use func throughout the exercises.

Exercise:
1) Using each of the three methods described above, define the function func(x,y)=sin(x^2+y^3) and evaluate at (x,y)=(sqrt(Pi)/4,0).

\[(x, y) \rightarrow \sin(x^2 + y^3)\]

\[\left(\frac{\sqrt{\pi}}{4}, 0\right)\]

\[f := (x, y) \rightarrow x^2 + y^3;\]

\[h := (x, y) \rightarrow x^2 + y^3;\]

\[h(k, m);\]

\[k^2 + m^3\]

(1.9)

Producing Slice Curves of functions

When we did cross sections of a graph before, we connected a function of two variables with a family of functions in one variables. This can be done by creating new functions. It can also be done by using a constant to fill in one of the variables.

\[f := (x, y) \rightarrow x^2 + y^3 - x \cdot y;\]

\[x0 := 1;\]

\[y0 := 2;\]

\[f1 := x \rightarrow f(x, y0);\]

\[f2 := y \rightarrow f(x0, y);\]

\[plot([f1(t), f2(t)], t=-2..2, color=[red, green], legend =\]
We can also plot a family of slices

```maple
> f := (x,y) -> x^2 + y^3 - x*y;
plot([f(x,-1), f(x,0.5), f(x,1)], x=-3..3,
     color=[red,green,blue], legend=["f(x,-1)", "f(x,0.5)", "f(x,1)"]);
```

\[
f := (x,y) \rightarrow x^2 + y^3 - xy
\]
Exercise:
2) Plot the slice of the function \( f(x,y) \), which you defined in exercise 1 above, with \( y = 2 \).

Computing Partial Derivatives

The partial derivative is computed by taking the derivative of a function of two variables, treating one of the variables as a constant.

In Maple, this is done with the `diff` command. It should be noted that the `diff` command has an "inert" version, `Diff`. Applying both commands we see that while Maple uses the same command for the derivative and the partial derivative, it uses a different symbol for the partial derivative.

```maple
> f := (x, y) -> x^2 + y^3 - x*y;
f := (x, y) -> x^2 + y^3 - x*y

> Dfx := diff(f(x,y),x);
Dfx := diff(f(x,y),x)

> Df1x := diff(f(x,y0),x);
Df1x := diff(f(x,y0),x)

> Diff(f,x);
Diff(f,x)

> Diff(f1,x);
Diff(f1,x)

> Diff(f(x,y0),x);
Diff(f(x,y0),x)

f := (x, y) \rightarrow x^2 + y^3 - x y

x^2 + y^3 - x y
\[ Dfx := 2x - y \]
\[ Df1x := 2x - 2 \]
\[ Df1xa := 2x - 2 \]
\[ \frac{\partial}{\partial x} (x^2 + y^3 - xy) \]
\[ \frac{d}{dx} (x^2 + 8 - 2x) \]
\[ \frac{d}{dx} (x^2 + 8 - 2x) \]  

(3.1)

> \textbf{Diff}(f(x,y),x) = \textbf{diff}(f(x,y),x) ;
\[ \frac{\partial}{\partial x} (x^2 + y^3 - xy) = 2x - y \]  

(3.2)

We can do the same computations in 2D math mode or by using the Expression palette.

\[ Dfy := \frac{\partial}{\partial y} f(x,y) ; \]
\[ 3y^2 - x \]  

(3.3)

Exercise:
3) Compute the partial derivatives of the function func(x,y), which you defined in exercise 1 above, both with respect to x and with respect to y.

> 2 = 3 ;
\[ 2 = 3 \]  

(3.4)

Finally, we would like to evaluate an expression at specific values. We use the \textbf{eval} command for this. The syntax is
\[ \textbf{eval}(\text{thing to be evaluated}, \text{set of values to be used}); \]
This can be done either by typing or using the palette. To get a decimal representation we use the \textbf{evalf} command.

> \textbf{Dfy} ;
\[ 3y^2 - x \]  

(3.5)

\[ Dfy \bigg|_{y=2} \]
\[ 12 - x \]  

(3.6)

\[ Dfy ; \]
\[ 3y^2 - x \]  

(3.7)
\begin{align*}
\left. Df_y \right|_{x=2} &= 3y^2 - 2 \\
\left( \left. Df_y \right|_{x=2} \right)_{y=3} &= 25 \\
\sin \left( \frac{1}{3} \pi \right) &= \frac{1}{2} \sqrt{3} \\
\text{evalf} \left( \sin \left( \frac{1}{3} \pi \right), 500 \right) &= 0.86602540378438646763723170752936183471402626905190314027903489725966508 \\
& 4544000185405730933786242878378130707077033515149849725474994762394058 \\
& 277560471868242640466159511527910339874100505054233746163250765617163345 \\
& 1661443325336127334460918985613523565830183930794009524993268689929694 \\
& 7338251737532880253783091740648030504738010935951625415729147619799164 \\
& 9889491225414435723191645867361208199229392769883397903190917683305542 \\
& 1586890447189158051044152762450835011760355572144347995478182898543584 \\
& 24903645 \\
\end{align*}

**Exercise:**

4) Evaluate both func and its partial derivative with respect to y at (2,3).