EXPLORING LENGTH WITH GAP

Recall that the length r of a permutation p is the smallest number of elements $s_1, s_2, \dots, s_r \in S = \{(1,2), (2,3), \dots, (n-1, n)\}$ such that $s_1 s_2 \dots s_r = p$.

Take the cycle $(1,2,3)$. You know where it send $1, 2, 3$. You also know where is sends $4, 5, 6, \dots$. (Note that the cycle $(1,2,3)$ does not have to be in S_3, it could be in S_n for any $n \geq 3$.)

1.1 How much disorder is there is the set $1, 2, \dots n$ after you use $(1,2,3)$. In other words how many pairs $i < j$ are there with $(1,2,3)(i) > (1,2,3)(j)$. Try this for $n = 3, 4, 5, \dots$. Now find the length of $(1,2,3)$. Do you have a conjecture?

You might want to look at the function `Movement(`σ`, n)`. If σ is a permutation and n an integer, it shows you all the $\sigma(i)$ in order.

1.2 Try this same problem for other 3-cyles and other values of n. Do you have a conjecture? (Remember the first sentence in this assignment. It seems sort of arbitrary, doesn't it.)

1.3 Try the same thing with the 4-cycle $(1,2,3,4)$, then with other 4-cycles. Do you have another conjecture?

Once you have gotten used to the counting, you can use the function `CountMovement` . It works the same as `Movement`, but it counts for you.

1.5 Create a conjecture about the number of pairs moved by an n-cycle. Randomly test your conjecture.