

The Least Square Worksheet

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In this worksheet we are considering a set of 10 ordered pairs representing atmospheric pressure (P) and weather temperature (T). The data are

P: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

T: 222, 227, 223, 233, 244, 253, 260, 266, 270, 266

```
> with(LinearAlgebra):  
> A := Matrix(10,2,[[1,1],[1,2],[1,3],[1,4],[1,5],[1,6],[1,7],[1,8],[1,9],[1,10]]);
```

$$A := \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \\ 1 & 6 \\ 1 & 7 \\ 1 & 8 \\ 1 & 9 \\ 1 & 10 \end{bmatrix} \quad (1)$$

```
> B := Multiply((Transpose(A)), A);
```

$$B := \begin{bmatrix} 10 & 55 \\ 55 & 385 \end{bmatrix} \quad (2)$$

```
> BI := B^(-1);
```

$$BI := \begin{bmatrix} \frac{7}{15} & \frac{-1}{15} \\ \frac{-1}{15} & \frac{2}{165} \end{bmatrix} \quad (3)$$

```
> TI := Vector[column](< 222 ,227 ,223 ,233 ,244 ,253 ,260 ,266 ,270 ,266 >);
```

$$TI := \begin{bmatrix} 222 \\ 227 \\ 223 \\ 233 \\ 244 \\ 253 \\ 260 \\ 266 \\ 270 \\ 266 \end{bmatrix} \quad (4)$$

> **AT2 :=Transpose(A).TI;**

$$AT2 := \begin{bmatrix} 2464 \\ 14053 \end{bmatrix} \quad (5)$$

> **LeastSQanswer1 :=BI.AT2;**

$$LeastSQanswer1 := \begin{bmatrix} 213 \\ \frac{334}{55} \end{bmatrix} \quad (6)$$

Thus the linear least square solution if ;

$$f := x \rightarrow 213 + \frac{334}{55} x \quad (7)$$

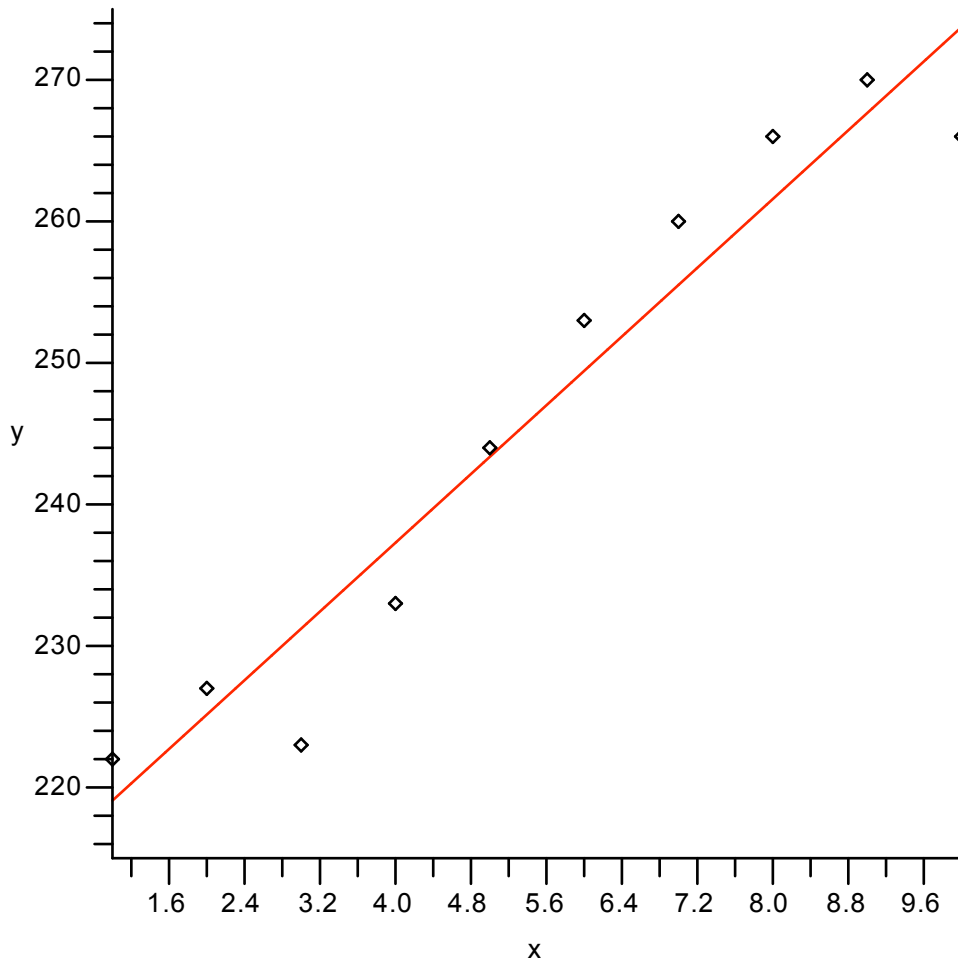
> **f := x -> LeastSQanswer1[2]*x^(2-1) + LeastSQanswer1[1]*x^(1-1);**
 $f := x \rightarrow LeastSQanswer1_2 x + LeastSQanswer1_1$ (8)

> **with(plots):**

> **Linearfit :=plot(f(x),x=1..10,y=215..275);**
 $Linearfit := INTERFACE_PLOT(...)$ (9)

> **Inidata := pointplot({[1,222],[2,227],[3,223],[4,233],[5,244],[6,253],[7,260],[8,266],[9,270],[10,266]});**
 $Inidata := INTERFACE_PLOT(...)$ (10)

> **display({Inidata, Linearfit});**



```
> V :=VandermondeMatrix([1,2,3,4,5,6,7,8,9,10]);
```

$$V := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 & 16 & 32 & 64 & 128 & 256 & 512 \\ 1 & 3 & 9 & 27 & 81 & 243 & 729 & 2187 & 6561 & 19683 \\ 1 & 4 & 16 & 64 & 256 & 1024 & 4096 & 16384 & 65536 & 262144 \\ 1 & 5 & 25 & 125 & 625 & 3125 & 15625 & 78125 & 390625 & 1953125 \\ 1 & 6 & 36 & 216 & 1296 & 7776 & 46656 & 279936 & 1679616 & 10077696 \\ 1 & 7 & 49 & 343 & 2401 & 16807 & 117649 & 823543 & 5764801 & 40353607 \\ 1 & 8 & 64 & 512 & 4096 & 32768 & 262144 & 2097152 & 16777216 & 134217728 \\ 1 & 9 & 81 & 729 & 6561 & 59049 & 531441 & 4782969 & 43046721 & 387420489 \\ 1 & 10 & 100 & 1000 & 10000 & 100000 & 1000000 & 10000000 & 100000000 & 1000000000 \end{bmatrix} \quad (11)$$

```
> A1 :=SubMatrix(V,1..10,1..4);
```

$$A1 := \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \\ 1 & 6 & 36 & 216 \\ 1 & 7 & 49 & 343 \\ 1 & 8 & 64 & 512 \\ 1 & 9 & 81 & 729 \\ 1 & 10 & 100 & 1000 \end{bmatrix} \quad (12)$$

> **B1 :=Multiply((Transpose(A1)),A1);**

$$B1 := \begin{bmatrix} 10 & 55 & 385 & 3025 \\ 55 & 385 & 3025 & 25333 \\ 385 & 3025 & 25333 & 220825 \\ 3025 & 25333 & 220825 & 1978405 \end{bmatrix} \quad (13)$$

> **BI1 :=B1^(-1);**

$$BI1 := \begin{bmatrix} \frac{113}{30} & \frac{-95}{36} & \frac{1}{2} & \frac{-1}{36} \\ \frac{-95}{36} & \frac{65365}{30888} & \frac{-50}{117} & \frac{761}{30888} \\ \frac{1}{2} & \frac{-50}{117} & \frac{103}{1144} & \frac{-5}{936} \\ \frac{-1}{36} & \frac{761}{30888} & \frac{-5}{936} & \frac{5}{15444} \end{bmatrix} \quad (14)$$

> **BT2 :=Transpose(A1).TI;**

$$BT2 := \begin{bmatrix} 2464 \\ 14053 \\ 100307 \\ 796321 \end{bmatrix} \quad (15)$$

> **LeastSQanswer3 :=BI1.BT2;**

$$\text{LeastSQanswer3} := \begin{bmatrix} \frac{6907}{30} \\ -\frac{53069}{5148} \\ \frac{6401}{1716} \\ -\frac{301}{1287} \end{bmatrix} \quad (16)$$

These are the coefficients of the cubic least square solution. That is

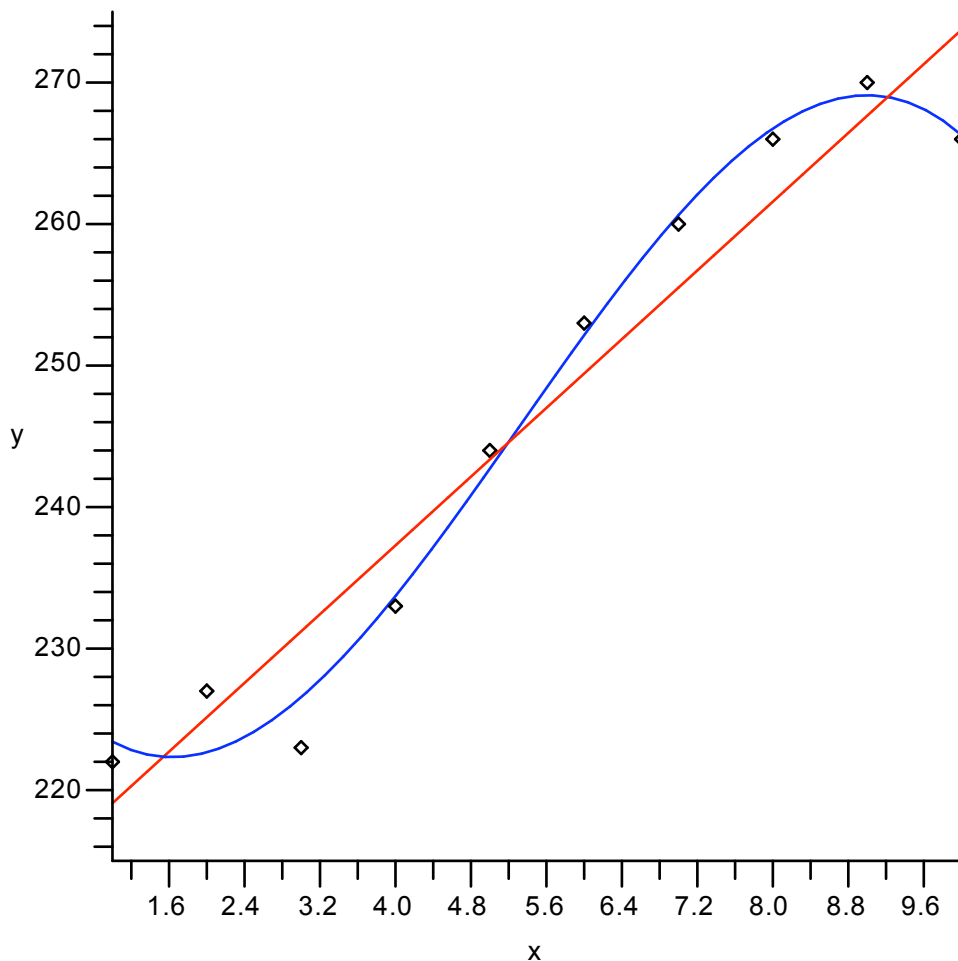
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> g := x -> (6907/30 + (-53069/5148)*x) + ((6401/1716)*x^2) + ((-301/1287)*x^3);
```

$$g := x \rightarrow \frac{6907}{30} - \frac{53069}{5148}x + \frac{6401}{1716}x^2 - \frac{301}{1287}x^3 \quad (17)$$

```
> cubefit := plot(g(x), x=1..10, y=215..275, color=blue);
cubefit := INTERFACE_PLOT(...)
```

(18)

```
> display({Linearfit, cubefit, Inidata});
```



```
> A2 :=SubMatrix(V,1..10,1..7);
```

$$A2 := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 & 16 & 32 & 64 \\ 1 & 3 & 9 & 27 & 81 & 243 & 729 \\ 1 & 4 & 16 & 64 & 256 & 1024 & 4096 \\ 1 & 5 & 25 & 125 & 625 & 3125 & 15625 \\ 1 & 6 & 36 & 216 & 1296 & 7776 & 46656 \\ 1 & 7 & 49 & 343 & 2401 & 16807 & 117649 \\ 1 & 8 & 64 & 512 & 4096 & 32768 & 262144 \\ 1 & 9 & 81 & 729 & 6561 & 59049 & 531441 \\ 1 & 10 & 100 & 1000 & 10000 & 100000 & 1000000 \end{bmatrix}$$

(19)

```
> B2 :=Multiply((Transpose(A2)),A2);
```

```
B2 := [10, 55, 385, 3025, 25333, 220825, 1978405], [55, 385, 3025, 25333, 220825, 1978405,
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(20)

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18080425], [385, 3025, 25333, 220825, 1978405, 18080425, 167731333], [3025, 25333, 220825, 1978405, 18080425, 167731333, 1574304985], [25333, 220825, 1978405, 18080425, 167731333, 1574304985, 14914341925], [220825, 1978405, 18080425, 167731333, 1574304985, 14914341925, 142364319625], [1978405, 18080425, 167731333, 1574304985,
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14914341925, 142364319625, 1367428536133]
```

```
> B13 :=B2^(-1);
```

$$B13 := \left[\frac{2416}{15}, \frac{-44933}{150}, \frac{696241}{3600}, \frac{-27979}{480}, \frac{12859}{1440}, \frac{-539}{800}, \frac{143}{7200} \right], \left[\frac{-44933}{150}, \frac{1460973269}{2574000} \right], \quad (21)$$

$$\begin{aligned} & \left[\frac{-115861961}{312000}, \frac{465447311}{4118400}, \frac{-2176669}{124800}, \frac{190571}{144000}, \frac{-627}{16000} \right], \left[\frac{696241}{3600}, \frac{-115861961}{312000} \right], \\ & \left[\frac{30387408037}{123552000}, \frac{-37741969}{499200}, \frac{52844411}{4492800}, \frac{-6726401}{7488000}, \frac{46207}{1728000} \right], \left[\frac{-27979}{480}, \frac{465447311}{4118400} \right], \\ & \left[\frac{-37741969}{499200}, \frac{51501253}{2196480}, \frac{-244607}{66560}, \frac{845887}{2995200}, \frac{-649}{76800} \right], \left[\frac{12859}{1440}, \frac{-2176669}{124800}, \frac{52844411}{4492800} \right], \\ & \left[\frac{-244607}{66560}, \frac{1041887}{1797120}, \frac{-134189}{2995200}, \frac{931}{691200} \right], \left[\frac{-539}{800}, \frac{190571}{144000}, \frac{-6726401}{7488000}, \frac{845887}{2995200} \right], \\ & \left[\frac{-134189}{2995200}, \frac{5789}{1664000}, \frac{-121}{1152000} \right], \left[\frac{143}{7200}, \frac{-627}{16000}, \frac{46207}{1728000}, \frac{-649}{76800}, \frac{931}{691200}, \frac{-121}{1152000} \right], \end{aligned}$$

$$\left[\frac{11}{3456000} \right]$$

```
> BT3 :=Transpose(A2).TI;
```

$$BT3 := \begin{bmatrix} 2464 \\ 14053 \\ 100307 \\ 796321 \\ 6707219 \\ 58659433 \\ 526556267 \end{bmatrix}$$

(22)

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> LeastSQanswer6 :=BI3.BT3;
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(23)

$$LeastSQanswer6 := \begin{bmatrix} \frac{1613}{10} \\ \frac{3462221}{28600} \\ -\frac{84915221}{1029600} \\ \frac{274849}{10560} \\ -\frac{30347}{7488} \\ \frac{19181}{62400} \\ -\frac{131}{14400} \end{bmatrix} \quad (23)$$

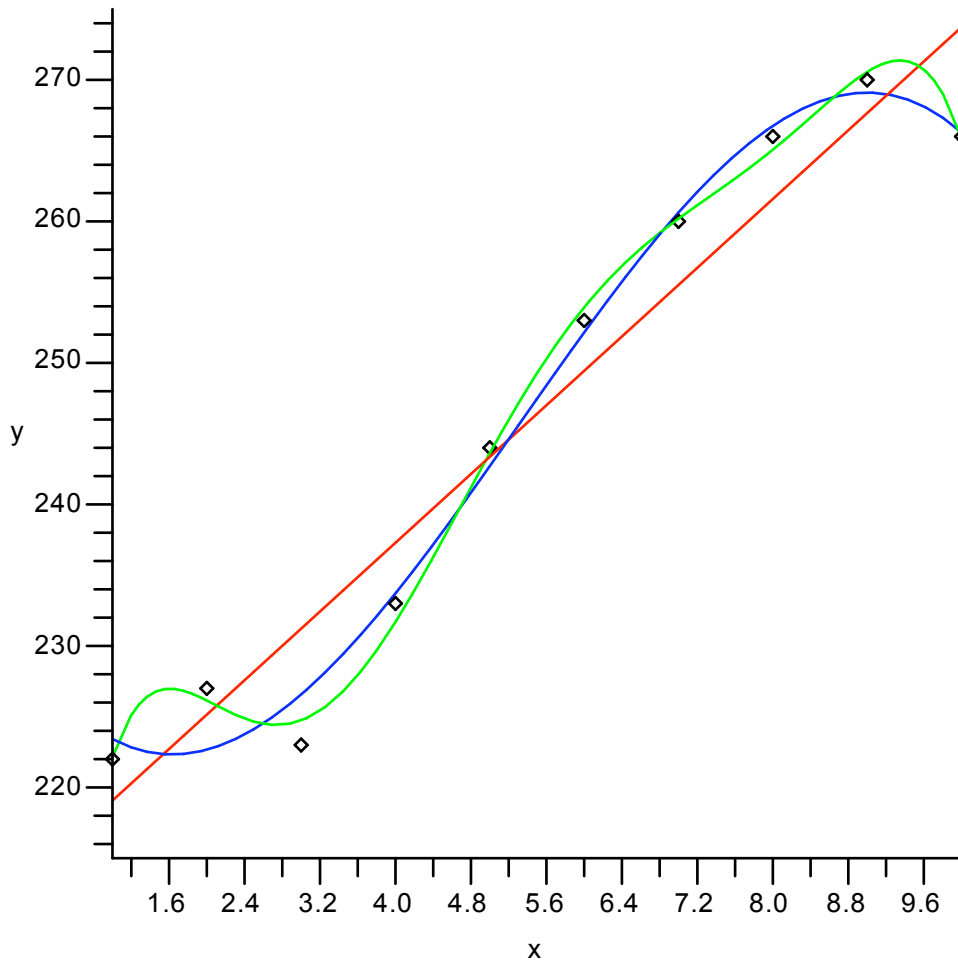
These are the coefficients of the 6th order polynomial least square solution. That is

$$h := x \rightarrow \frac{1613}{10} + \frac{3462221}{28600} x - \frac{84915221}{1029600} x^2 + \frac{274849}{10560} x^3 - \frac{30347}{7488} x^4 + \frac{19181}{62400} x^5 - \frac{131}{14400} x^6 \quad (24)$$

$$> \text{hexfit} := \text{plot}(h(x), x=1..10, y=215..275, \text{color}=\text{green});$$

$$\text{hexfit} := \text{INTERFACE_PLOT}(\dots) \quad (25)$$

$$> \text{display}(\{\text{Linearfit}, \text{cubefit}, \text{Inidata}, \text{hexfit}\});$$



```
> B3 :=Multiply((Transpose(V)),V);
```

```
B3 := [10, 55, 385, 3025, 25333, 220825, 1978405, 18080425, 167731333, 1574304985], [55, (26)
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385, 3025, 25333, 220825, 1978405, 18080425, 167731333, 1574304985, 14914341925], [385,
```

3025, 25333, 220825, 1978405, 18080425, 167731333, 1574304985, 14914341925,
142364319625], [3025, 25333, 220825, 1978405, 18080425, 167731333, 1574304985,
14914341925, 142364319625, 1367428536133], [25333, 220825, 1978405, 18080425,
167731333, 1574304985, 14914341925, 142364319625, 1367428536133, 13202860761145], [
220825, 1978405, 18080425, 167731333, 1574304985, 14914341925, 142364319625,
1367428536133, 13202860761145, 128037802953445], [1978405, 18080425, 167731333,
1574304985, 14914341925, 142364319625, 1367428536133, 13202860761145,
128037802953445, 1246324856379625], [18080425, 167731333, 1574304985, 14914341925,
142364319625, 1367428536133, 13202860761145, 128037802953445, 1246324856379625,
12170706132009733], [167731333, 1574304985, 14914341925, 142364319625,
1367428536133, 13202860761145, 128037802953445, 1246324856379625,
12170706132009733, 119179318935377305], [1574304985, 14914341925, 142364319625,
1367428536133, 13202860761145, 128037802953445, 1246324856379625,

12170706132009733, 119179318935377305, 1169842891165484965]

> **BI4 :=B3⁽⁻¹⁾;**

$$BI4 := \left[184755, \frac{-316154003}{630}, \frac{78227545}{144}, \frac{-1425155303}{4536}, \frac{62539477}{576}, \frac{-203088457}{8640}, \frac{925925}{288} \right], (27)$$

$$\begin{aligned} & \left[\frac{-1631201}{6048}, \frac{2431}{192}, \frac{-46189}{181440} \right] \left[\frac{-316154003}{630}, \frac{4330231692391}{3175200}, \frac{-15315434761}{10368}, \right. \\ & \frac{19543176541471}{22861440}, \frac{-61294153129}{207360}, \frac{1394099727427}{21772800}, \frac{-1271867179}{145152}, \frac{11208585641}{15240960}, \\ & \left. \frac{-10026907}{290304}, \frac{317641879}{457228800} \right] \left[\frac{78227545}{144}, \frac{-15315434761}{10368}, \frac{464822181637}{290304}, \right. \\ & \frac{-84796491345577}{91445760}, \frac{1330738013759}{4147200}, \frac{-242305672741}{3483648}, \frac{3950106611}{414720}, \frac{-6966419273}{8709120}, \\ & \left. \frac{1091192737}{29030400}, \frac{-55335841}{73156608} \right] \left[\frac{-1425155303}{4536}, \frac{19543176541471}{22861440}, \frac{-84796491345577}{91445760}, \right. \\ & \frac{2211663567269299}{4115059200}, \frac{-1945234715281}{10450944}, \frac{158246096437}{3919104}, \frac{-144571917919}{26127360}, \frac{79731182999}{171460800}, \\ & \left. \frac{-1599550667}{73156608}, \frac{362324611}{823011840} \right] \left[\frac{62539477}{576}, \frac{-61294153129}{207360}, \frac{1330738013759}{4147200}, \right. \\ & \frac{-1945234715281}{10450944}, \frac{71346107833}{1105920}, \frac{-27881756903}{1990656}, \frac{1770264223}{921600}, \frac{-160784195}{995328}, \\ & \left. \frac{25216499}{3317760}, \frac{-32005831}{209018880} \right] \left[\frac{-203088457}{8640}, \frac{1394099727427}{21772800}, \frac{-242305672741}{3483648}, \right. \end{aligned}$$

$$\begin{aligned}
& \frac{158246096437}{3919104}, \frac{-27881756903}{1990656}, \frac{227180317559}{74649600}, \frac{-415726597}{995328}, \frac{918384533}{26127360}, \\
& \frac{-115302473}{69672960}, \frac{52297817}{1567641600} \Big[\frac{925925}{288}, \frac{-1271867179}{145152}, \frac{3950106611}{414720}, \frac{-144571917919}{26127360}, \\
& \frac{1770264223}{921600}, \frac{-415726597}{995328}, \frac{15860537}{276480}, \frac{-16829527}{3483648}, \frac{1887743}{8294400}, \frac{-95953}{20901888} \Big] \Big[\frac{-1631201}{6048}, \\
& \frac{11208585641}{15240960}, \frac{-6966419273}{8709120}, \frac{79731182999}{171460800}, \frac{-160784195}{995328}, \frac{918384533}{26127360}, \frac{-16829527}{3483648}, \\
& \frac{372276047}{914457600}, \frac{-133705}{6967296}, \frac{106249}{274337280} \Big] \Big[\frac{2431}{192}, \frac{-10026907}{290304}, \frac{1091192737}{29030400}, \frac{-1599550667}{73156608}, \\
& \frac{25216499}{3317760}, \frac{-115302473}{69672960}, \frac{1887743}{8294400}, \frac{-133705}{6967296}, \frac{1001}{1105920}, \frac{-26741}{1463132160} \Big] \Big[\frac{-46189}{181440}, \\
& \frac{317641879}{457228800}, \frac{-55335841}{73156608}, \frac{362324611}{823011840}, \frac{-32005831}{209018880}, \frac{52297817}{1567641600}, \frac{-95953}{20901888}, \frac{106249}{274337280} \Big]
\end{aligned}$$

$$\left[\frac{-26741}{1463132160}, \frac{2431}{6584094720} \right]$$

> **BT4 :=Transpose(V).TI;**

$$BT4 := \begin{bmatrix} 2464 \\ 14053 \\ 100307 \\ 796321 \\ 6707219 \\ 58659433 \\ 526556267 \\ 4817586001 \\ 44721249059 \\ 419889190393 \end{bmatrix} \quad (28)$$

> **LeastSQanswer9 :=BI4.BT4;**

$$LeastSQanswer9 := \begin{bmatrix} -143 \\ \frac{2257253}{2520} \\ -\frac{8606929}{10080} \\ \frac{19586951}{45360} \\ -\frac{251551}{1920} \\ \frac{435707}{17280} \\ -\frac{2987}{960} \\ \frac{14447}{60480} \\ -\frac{419}{40320} \\ \frac{71}{362880} \end{bmatrix} \quad (29)$$

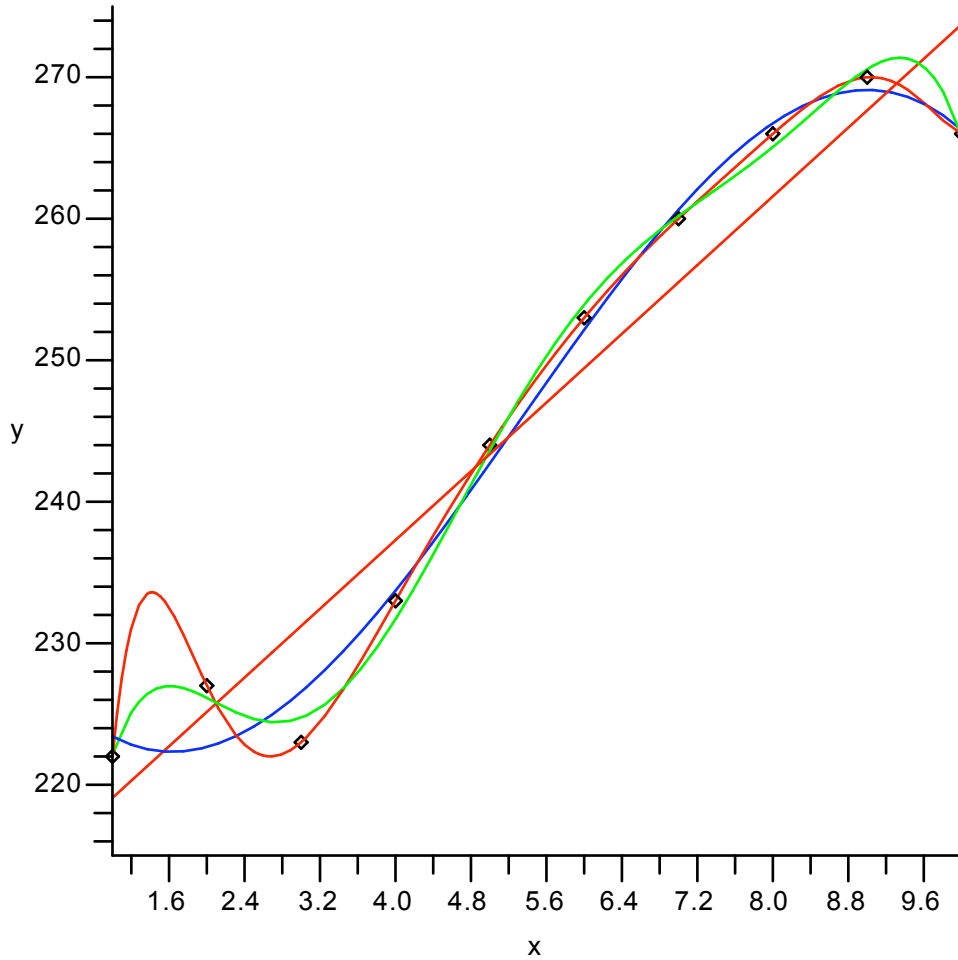
These are the coefficients of the 9th order polynomial least square solution. That is

> **F :=x->(-143+((2257253/2520)*x)+((-8606929/10080)*x^2)+((19586951/45360)*x^3)+((-251551/1920)*x^4)+((435707/17280)*x^5)+((-2987/960)*x^6)+((14447/60480)*x^7)+((-419/40320)*x^8)+((71/362880)*x^9));**

$$F := x \rightarrow -143 + \frac{2257253}{2520} x - \frac{8606929}{10080} x^2 + \frac{19586951}{45360} x^3 - \frac{251551}{1920} x^4 + \frac{435707}{17280} x^5 - \frac{2987}{960} x^6 + \frac{14447}{60480} x^7 - \frac{419}{40320} x^8 + \frac{71}{362880} x^9 \quad (30)$$

```
> ninefit := plot(F(x), x=1..10, y=215..275, color=red);
      ninefit := INTERFACE_PLOT(...)
```

```
> display({Linearfit, cubefit, Inidata, hexfit, ninefit});
```



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```