

# Image of a Linear Transformation

Worksheet by Russell Blyth

```
> restart: with(LinearAlgebra): with(plots): with(plottools):  
Warning, the name changecoords has been redefined  
Warning, the assigned name arrow now has a global binding
```

## Outline

The basic objectives are:

- 1) Investigate the image of a particular linear transformation.
- 2) Graphically investigate the null space of the linear transformation.

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## Working with the image of a linear transformation

Define a random 2 x 3 matrix of rank 1.

```
> A := RandomMatrix(1,3,generator=rand(-10..10));  
A := <A,rand(-10..10)()*Row(A,1)>;
```

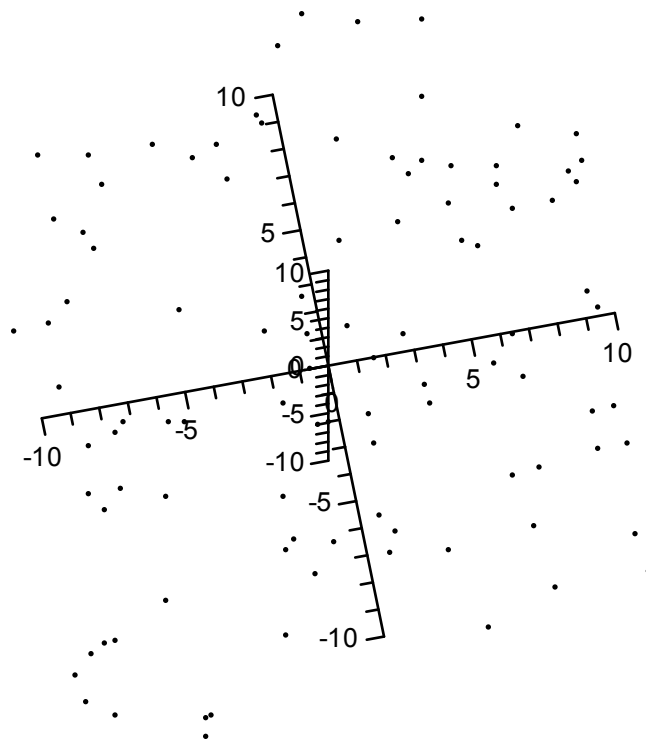
$$A := \begin{bmatrix} 4 & 2 & -5 \\ 4 & 2 & -5 \\ -36 & -18 & 45 \end{bmatrix} \quad (2.1)$$

Interpret A as a matrix that represents a linear transformation T from  $\mathbb{R}^3$  to  $\mathbb{R}^2$

What is the dimension of the image of T? We investigate by creating 100 random points in  $\mathbb{R}^3$  and finding and plotting the images of these 100 points under multiplication on the left by the matrix A.

First the 100 points:

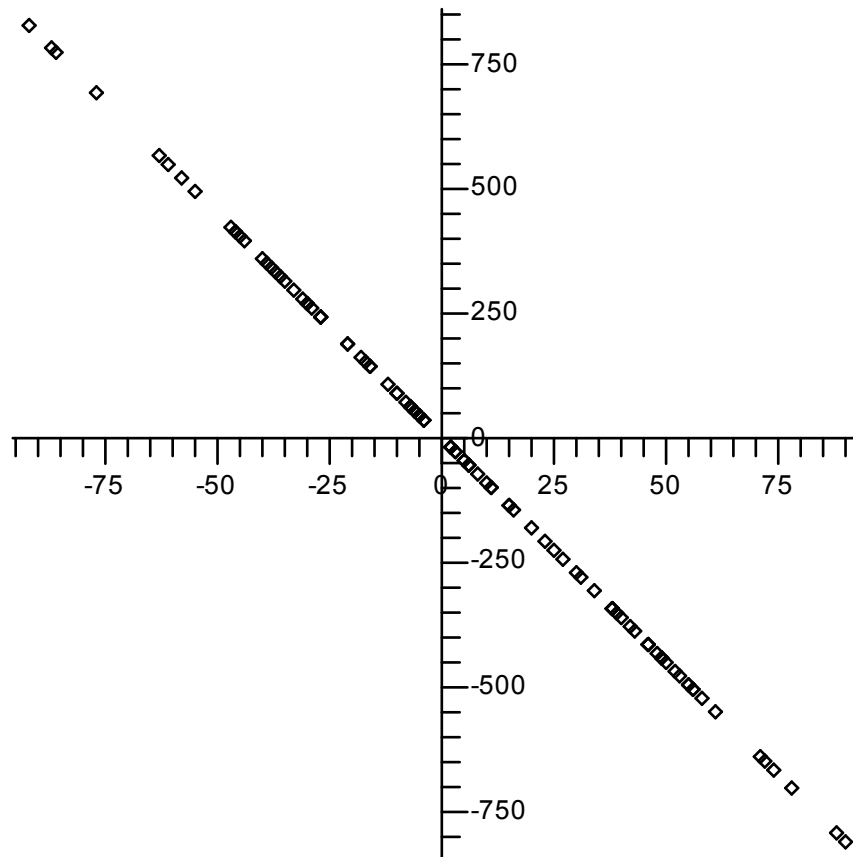
```
> setofpoints :=  
{seq(Vector(3, [rand(-10..10)(),rand(-10..10)(),rand(-10..10)()])  
,  
i=1..100)  
};  
pointplot3d(setofpoints,view=[-10..10,-10..10,-10..10],  
axes=normal,color=black);
```



Rotate the plot to see that the points are spread around in  $\mathbb{R}^3$ .

Next, plot the images of these points:

```
> setofimages := {seq(A.setofpoints[i],i=1..100)} minus {0};  
pointplot(setofimages,color=black);
```



Questions: why do you see only a line segment for the image rather than a whole line? How can you get more of the line? What is the slope of the line (note that the line does not have the slope it appears to have due to the scaling of the axes).

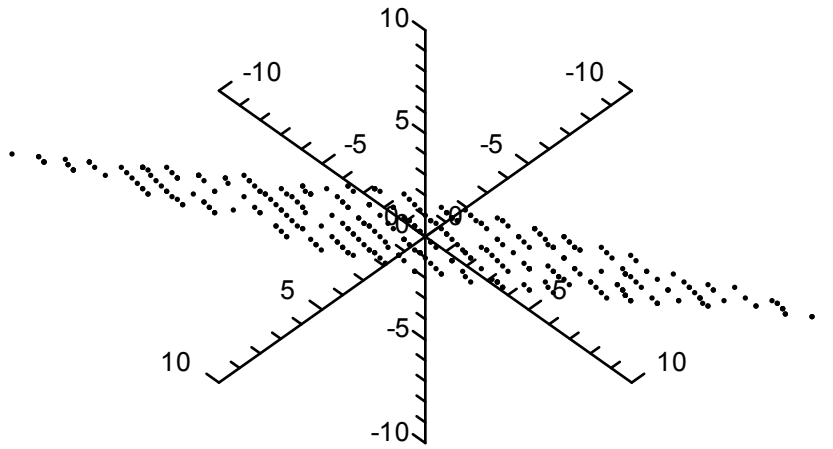
Next, look at the null space of A.

```
> nullbasis:=NullSpace(A);
```

$$\text{nullbasis} := \left\{ \begin{bmatrix} \frac{5}{4} \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{bmatrix} \right\} \quad (2.2)$$

Plot a few points in the nullspace by computing some random linear combinations of the basis elements.

```
> nullpoints := {seq(rand(-10..10)()*nullbasis[1] +
    rand(-10..10)()*nullbasis[2],i=1..500)} minus {0}:
pointplot3d(nullpoints,view=[-10..10,-10..10,-10..10],
    axes=normal,color=black);
```



Rotate the plot to see that the null space is a plane, and hence has dimension 2.

Exercise:

- 1) Create a random  $3 \times 3$  matrix  $B$  of rank 2. Repeat the calculations and plots performed above for the matrix  $A$  for your matrix  $B$ . Note that  $B$  represents a linear transformation from  $\mathbb{R}^3$  to  $\mathbb{R}^3$ , so all plots will be 3D plots.

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