In-Class Demonstration: LU-Decomposition

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```
> restart: with(LinearAlgebra): with(plots): with(plottools):
Warning, the name changecoords has been redefined
Warning, the assigned name arrow now has a global binding
```

We start with a random augmented matrix. We then repeat the matrix so we can keep the computations stable.

```
> A := RandomMatrix(3,4,generator=rand(-5..5));
```

$$A := \begin{bmatrix} 4 & 5 & -4 & -4 \\ -2 & 2 & 5 & -3 \\ 3 & 4 & -4 & 5 \end{bmatrix} \quad (1)$$

```
> A := <<4 | -1 | -4 | -1>, <-1 | -4 | -2 | 3>, <4 | 4 | -3 | -3>>;
```

$$A := \begin{bmatrix} 4 & -1 & -4 & -1 \\ -1 & -4 & -2 & 3 \\ 4 & 4 & -3 & -3 \end{bmatrix} \quad (2)$$

We augment the matrix by the identity matrix to record row reductions, then do normal step by step Gaussian elimination.

```
> A1 := <A|IdentityMatrix(3)>;
```

$$A1 := \begin{bmatrix} 4 & 5 & -4 & -4 & 1 & 0 & 0 \\ -2 & 2 & 5 & -3 & 0 & 1 & 0 \\ 3 & 4 & -4 & 5 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

```
> A2 := RowOperation(A1,[2,1],1/2);
```

$$A2 := \begin{bmatrix} 4 & 5 & -4 & -4 & 1 & 0 & 0 \\ 0 & \frac{9}{2} & 3 & -5 & \frac{1}{2} & 1 & 0 \\ 3 & 4 & -4 & 5 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

```
> A3 := RowOperation(A2,[3,1],-1);
```

$$A3 := \begin{bmatrix} 4 & -1 & -4 & -1 & 1 & 0 & 0 \\ 0 & \frac{-17}{4} & -3 & \frac{11}{4} & \frac{1}{4} & 1 & 0 \\ 0 & 5 & 1 & -2 & -1 & 0 & 1 \end{bmatrix} \quad (5)$$

```
> A4 := RowOperation(A3,[3,2],20/17);
```

(6)

$$A4 := \begin{bmatrix} 4 & -1 & -4 & -1 & 1 & 0 & 0 \\ 0 & \frac{-17}{4} & -3 & \frac{11}{4} & \frac{1}{4} & 1 & 0 \\ 0 & 0 & \frac{-43}{17} & \frac{21}{17} & \frac{-12}{17} & \frac{20}{17} & 1 \end{bmatrix} \quad (6)$$

We keep track of the coefficients used, namely 1/4, -1, 20/17.

We next take the appropriate columns of the coefficient matrix of A to find U and B .
L is the inverse of B.

```
> U1 := DeleteColumn(A4,5..7);
  U := DeleteColumn(A4,4..7);
  B := DeleteColumn(A4,1..4);
```

$$U1 := \begin{bmatrix} 4 & -1 & -4 & -1 \\ 0 & \frac{-17}{4} & -3 & \frac{11}{4} \\ 0 & 0 & \frac{-43}{17} & \frac{21}{17} \end{bmatrix}$$

$$U := \begin{bmatrix} 4 & -1 & -4 \\ 0 & \frac{-17}{4} & -3 \\ 0 & 0 & \frac{-43}{17} \end{bmatrix}$$

$$B := \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{4} & 1 & 0 \\ \frac{-12}{17} & \frac{20}{17} & 1 \end{bmatrix} \quad (7)$$

```
> L := B^(-1);
```

$$L := \begin{bmatrix} 1 & 0 & 0 \\ \frac{-1}{4} & 1 & 0 \\ 1 & \frac{-20}{17} & 1 \end{bmatrix} \quad (8)$$

Note the the entries of L are simply the negatives of the coefficients used.
Note also that L times U1 gives us the original matrix A.

```
> L.U1;
  A;
```

$$\begin{bmatrix} 4 & -1 & -4 & -1 \\ -1 & -4 & -2 & 3 \\ 4 & 4 & -3 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -1 & -4 & -1 \\ -1 & -4 & -2 & 3 \\ 4 & 4 & -3 & -3 \end{bmatrix} \quad (9)$$

Now we look at solving the system of equations represented by A.

```
> C := Column(A,4);
```

$$C := \begin{bmatrix} -1 \\ 3 \\ -3 \end{bmatrix} \quad (10)$$

```
> S1 := LinearSolve(A);
```

$$S1 := \begin{bmatrix} \frac{-35}{43} \\ \frac{-13}{43} \\ \frac{-21}{43} \end{bmatrix} \quad (11)$$

We should get to the same solution if we solve the pair of triangular systems LY = C followed by UX = Y.

```
> S2 := LinearSolve(L,C);
```

$$S2 := \begin{bmatrix} -1 \\ \frac{11}{4} \\ \frac{21}{17} \end{bmatrix} \quad (12)$$

```
> S3 := LinearSolve(U,S2);
```

$$S3 := \begin{bmatrix} \frac{-35}{43} \\ \frac{-13}{43} \\ \frac{-21}{43} \end{bmatrix} \quad (13)$$

```
>
```