Algorithmic Gaussian Elimination


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> restart:
  with(LinearAlgebra):
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One of the strengths of Gaussian Elimination is that it can be done mechanically to a randomly chosen matrix. It is worthwhile to see that it works in this mode.

We will assume that the reduction can be done without any row swapping. We print out the result after full use of each pivot.

```
> rsize := 7: csize := 8:
> M1 := RandomMatrix(rsize, csize, generator = rand(10)):
  M[0] := M1:
  for i from 1 to rsize do:
  M[i] := M[i-1]:
  for j from i+1 to rsize do:
  M[i] := RowOperation(M[i],[j,i],-M[i][j,i]/M[i][i,i]);
  end do;
  end do;
```

$$M_1 := \begin{bmatrix} 9 & 1 & 4 & 0 & 2 & 8 & 1 & 1 \\ 2 & 4 & 3 & 3 & 4 & 7 & 6 & 0 \\ 1 & 3 & 1 & 1 & 2 & 1 & 5 & 3 \\ 9 & 3 & 3 & 6 & 4 & 0 & 4 & 5 \\ 2 & 7 & 8 & 8 & 4 & 2 & 0 & 8 \\ 7 & 9 & 6 & 2 & 6 & 2 & 7 & 3 \\ 0 & 6 & 0 & 5 & 7 & 3 & 0 & 3 \end{bmatrix}$$

$$M_2 := \begin{bmatrix} 9 & 1 & 4 & 0 & 2 & 8 & 1 & 1 \\ 0 & \frac{34}{9} & \frac{19}{9} & 3 & \frac{32}{9} & \frac{47}{9} & \frac{52}{9} & \frac{-2}{9} \\ 0 & \frac{26}{9} & \frac{5}{9} & 1 & \frac{16}{9} & \frac{1}{9} & \frac{44}{9} & \frac{26}{9} \\ 0 & 2 & -1 & 6 & 2 & -8 & 3 & 4 \\ 0 & \frac{61}{9} & \frac{64}{9} & 8 & \frac{32}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{70}{9} \\ 0 & \frac{74}{9} & \frac{26}{9} & 2 & \frac{40}{9} & \frac{-38}{9} & \frac{56}{9} & \frac{20}{9} \\ 0 & 6 & 0 & 5 & 7 & 3 & 0 & 3 \end{bmatrix}$$

$$M_3 := \begin{bmatrix} 9 & 1 & 4 & 0 & 2 & 8 & 1 & 1 \\ 0 & \frac{34}{9} & \frac{19}{9} & 3 & \frac{32}{9} & \frac{47}{9} & \frac{52}{9} & \frac{-2}{9} \\ 0 & 0 & \frac{-18}{17} & \frac{-22}{17} & \frac{-16}{17} & \frac{-66}{17} & \frac{8}{17} & \frac{52}{17} \\ 0 & 0 & \frac{-36}{17} & \frac{75}{17} & \frac{2}{17} & \frac{-183}{17} & \frac{-1}{17} & \frac{70}{17} \\ 0 & 0 & \frac{113}{34} & \frac{89}{34} & \frac{-48}{17} & \frac{-311}{34} & \frac{-180}{17} & \frac{139}{17} \\ 0 & 0 & \frac{-29}{17} & \frac{-77}{17} & \frac{-56}{17} & \frac{-265}{17} & \frac{-108}{17} & \frac{46}{17} \\ 0 & 0 & \frac{-57}{17} & \frac{4}{17} & \frac{23}{17} & \frac{-90}{17} & \frac{-156}{17} & \frac{57}{17} \end{bmatrix}$$

$$M_4 := \begin{bmatrix} 9 & 1 & 4 & 0 & 2 & 8 & 1 & 1 \\ 0 & \frac{34}{9} & \frac{19}{9} & 3 & \frac{32}{9} & \frac{47}{9} & \frac{52}{9} & \frac{-2}{9} \\ 0 & 0 & \frac{-18}{17} & \frac{-22}{17} & \frac{-16}{17} & \frac{-66}{17} & \frac{8}{17} & \frac{52}{17} \\ 0 & 0 & 0 & 7 & 2 & -3 & -1 & -2 \\ 0 & 0 & 0 & \frac{-13}{9} & \frac{-52}{9} & \frac{-64}{3} & \frac{-82}{9} & \frac{160}{9} \\ 0 & 0 & 0 & \frac{-22}{9} & \frac{-16}{9} & \frac{-28}{3} & \frac{-64}{9} & \frac{-20}{9} \\ 0 & 0 & 0 & \frac{13}{3} & \frac{13}{3} & 7 & \frac{-32}{3} & \frac{-19}{3} \end{bmatrix}$$

$$M_5 := \begin{bmatrix} 9 & 1 & 4 & 0 & 2 & 8 & 1 & 1 \\ 0 & \frac{34}{9} & \frac{19}{9} & 3 & \frac{32}{9} & \frac{47}{9} & \frac{52}{9} & \frac{-2}{9} \\ 0 & 0 & \frac{-18}{17} & \frac{-22}{17} & \frac{-16}{17} & \frac{-66}{17} & \frac{8}{17} & \frac{52}{17} \\ 0 & 0 & 0 & 7 & 2 & -3 & -1 & -2 \\ 0 & 0 & 0 & 0 & \frac{-338}{63} & \frac{-461}{21} & \frac{-587}{63} & \frac{1094}{63} \\ 0 & 0 & 0 & 0 & \frac{-68}{63} & \frac{-218}{21} & \frac{-470}{63} & \frac{-184}{63} \\ 0 & 0 & 0 & 0 & \frac{65}{21} & \frac{62}{7} & \frac{-211}{21} & \frac{-107}{21} \end{bmatrix}$$

$$M_6 := \begin{bmatrix} 9 & 1 & 4 & 0 & 2 & 8 & 1 & 1 \\ 0 & \frac{34}{9} & \frac{19}{9} & 3 & \frac{32}{9} & \frac{47}{9} & \frac{52}{9} & \frac{-2}{9} \\ 0 & 0 & \frac{-18}{17} & \frac{-22}{17} & \frac{-16}{17} & \frac{-66}{17} & \frac{8}{17} & \frac{52}{17} \\ 0 & 0 & 0 & 7 & 2 & -3 & -1 & -2 \\ 0 & 0 & 0 & 0 & \frac{-338}{63} & \frac{-461}{21} & \frac{-587}{63} & \frac{1094}{63} \\ 0 & 0 & 0 & 0 & 0 & \frac{-1008}{169} & \frac{-944}{169} & \frac{-1084}{169} \\ 0 & 0 & 0 & 0 & 0 & \frac{-99}{26} & \frac{-401}{26} & \frac{64}{13} \end{bmatrix}$$

(1)

$$M_7 := \begin{bmatrix} 9 & 1 & 4 & 0 & 2 & 8 & 1 & 1 \\ 0 & \frac{34}{9} & \frac{19}{9} & 3 & \frac{32}{9} & \frac{47}{9} & \frac{52}{9} & \frac{-2}{9} \\ 0 & 0 & \frac{-18}{17} & \frac{-22}{17} & \frac{-16}{17} & \frac{-66}{17} & \frac{8}{17} & \frac{52}{17} \\ 0 & 0 & 0 & 7 & 2 & -3 & -1 & -2 \\ 0 & 0 & 0 & 0 & \frac{-338}{63} & \frac{-461}{21} & \frac{-587}{63} & \frac{1094}{63} \\ 0 & 0 & 0 & 0 & 0 & \frac{-1008}{169} & \frac{-944}{169} & \frac{-1084}{169} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-83}{7} & \frac{505}{56} \end{bmatrix} \quad (1)$$

Re-execute the section above many times.

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Note 1 - If you try this with a matrix with a dimension greater than 10, Maple simply tells you it has a matrix rather than printing it out. The simplest way to see such a matrix is to double click it and to invoke the matrix browser. One can also change the size at which this happens can be shifted with an interface(rtablesize=n) command.

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Note 2 - You should note that the algorithm fails if a zero shows up at the wrong spot. Test by example to see how often this happens. Decide if you want to modify the demo to eliminate this problem or if you want to incorporate this into your teaching.

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