

Symmetries of an Equilateral Triangle

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This is a first draft of a worksheet for the beginning of Abstract Algebra

▼ Outline

1. Investigate the symmetries of an equilateral triangle visually using plottools.
2. Investigate the composition of two transformations visually using plottools.

▼ Symmetries of an Equilateral Triangle

▼ Define and plot an equilateral triangle

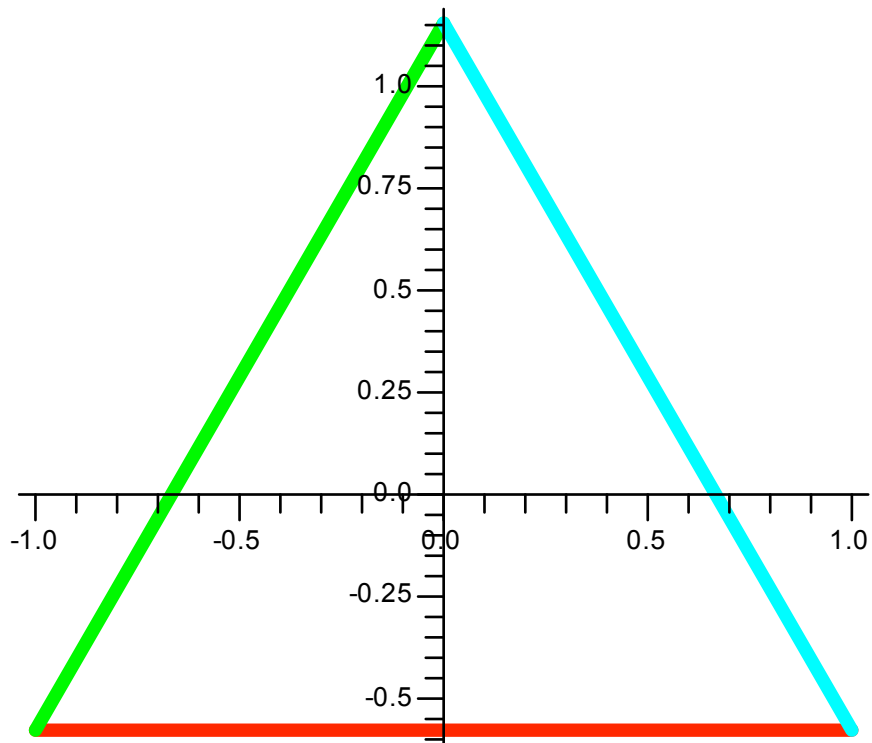
```
> restart: with(plots):with(plottools):  
Warning, the name changecoords has been redefined  
Warning, the assigned name arrow now has a global binding
```

The following define the sides of the equilateral triangle.

```
> L1 := line([1, -sqrt(3)/3], [0, 2*sqrt(3)/3], color = cyan,  
thickness = 5):  
L2 := line([0, 2*sqrt(3)/3], [-1, -sqrt(3)/3], color = green,  
thickness = 5):  
L3 := line([-1, -sqrt(3)/3], [1, -sqrt(3)/3], color = red,  
thickness = 5):
```

Next we display the triangle. (How can I get the same scale on both axes?)

```
> display(L1, L2, L3, scaling = constrained);
```

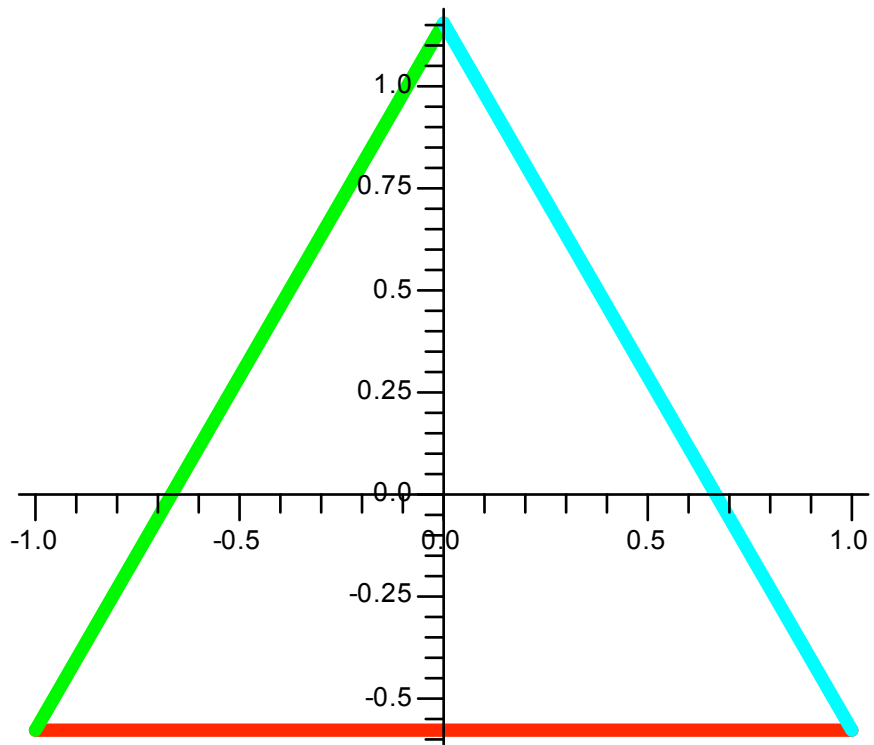


Name it so that we can deal with it as an object.

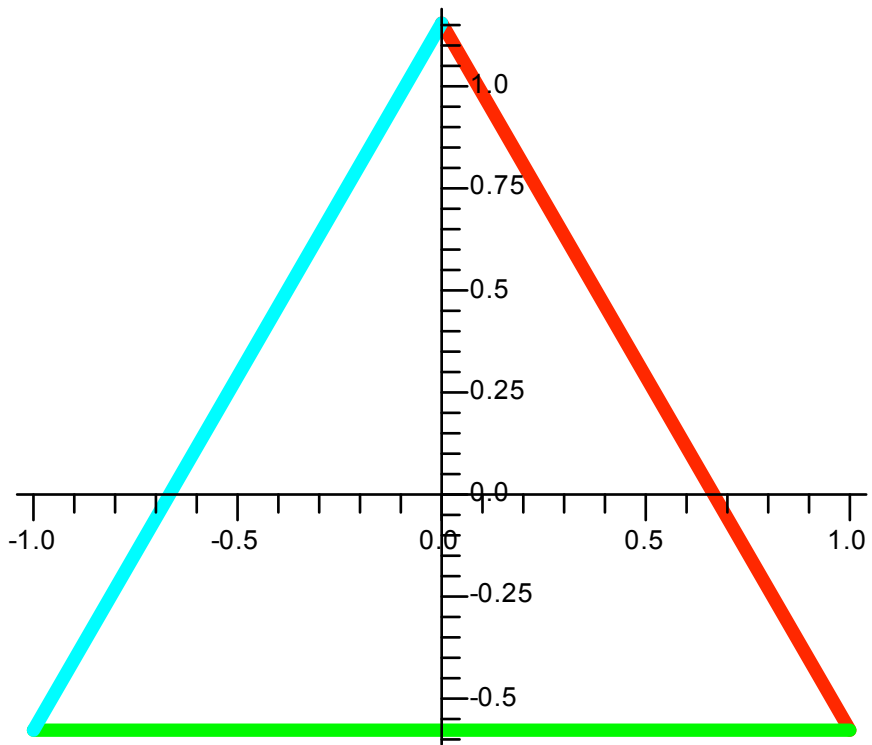
```
[ > ET := display(L1, L2, L3, scaling = constrained) :
```

▼ Rotations

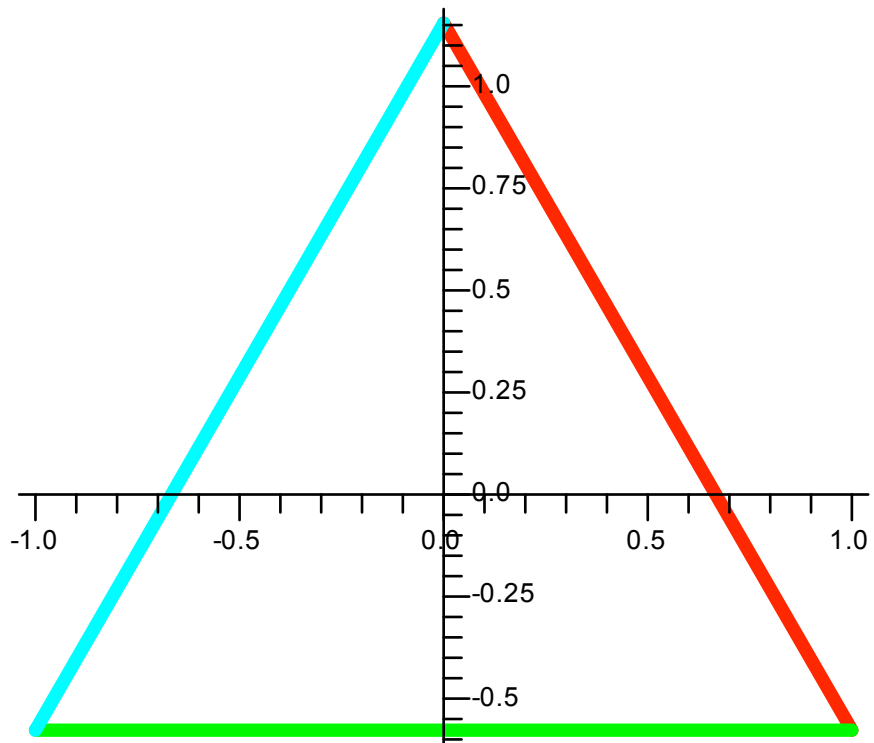
```
[ > R0 := rotate(ET, 0, [0, 0]) :  
display(ET) ;
```



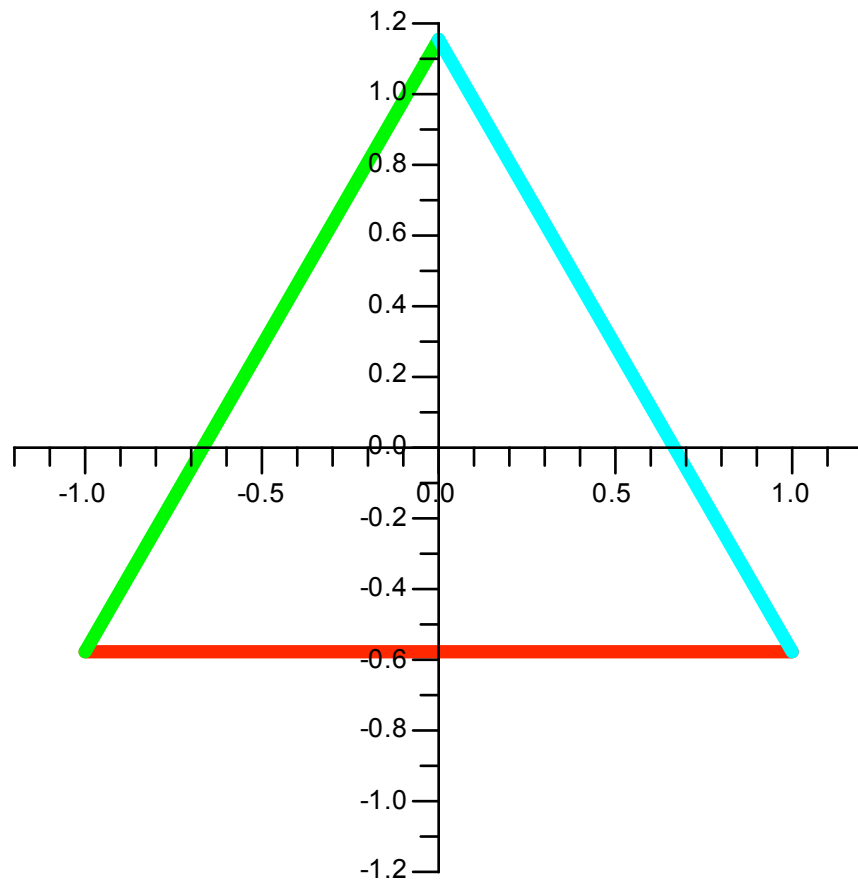
```
> R120 := rotate(ET, 2*Pi/3, [0, 0]):  
display(R120);
```



> R120;



```
>  
> display([seq(rotate(ET, (counta/20)*2*Pi/3, [0,0]), counta=0.  
.20)],  
          insequence = true);
```



```

> R240 := rotate(ET,4*Pi/3,[0,0]):
display(R240);
>
> R70 := rotate(ET,7*Pi/18,[0,0]):
display(R70);

```

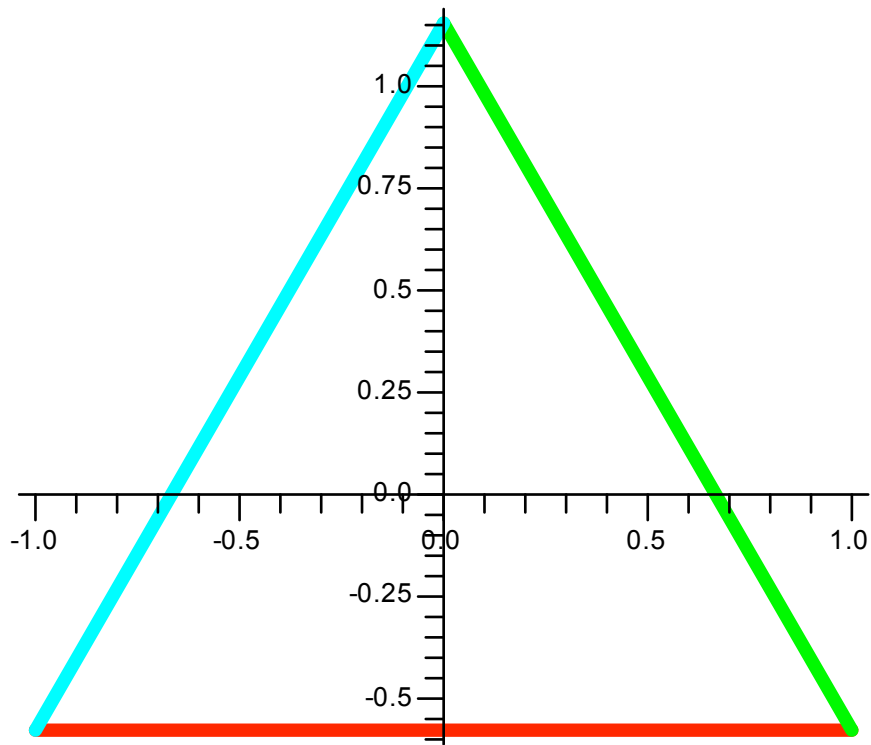
▼ Reflections

Reflect in the line containing the first altitude.

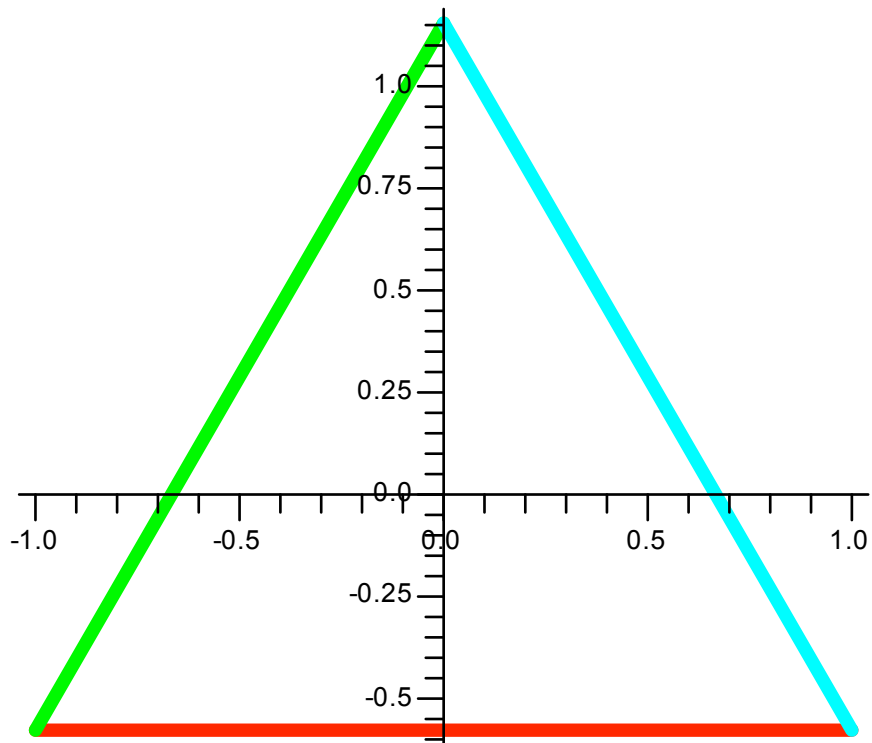
```

> A1 := reflect(ET,[0,2*sqrt(3)/3],[0,-sqrt(3)/3]):
display(A1);

```



```
> display([seq(display(line([1*(1-counta/10), -sqrt(3)/3], [0,
2*sqrt(3)/3], color = cyan, thickness = 5), line([0, 2*sqrt(3)
/3], [-1*(1-counta/10), -sqrt(3)/3], color = green, thickness
= 5), line([-1*(1-counta/10), -sqrt(3)/3], [1*(1-counta/10), -
sqrt(3)/3], color = red, thickness = 5), scaling=constrained),
counta=0..20)],
insequence = true);
```



Reflect in the line containing the second altitude.

```
[ > A2 := reflect(ET, [], []):
  display(A2);
```

```
[ >
```

Needs more work in controlling scale.

```
[ > A3 :=
```

```
[ >
```

▼ Composition of Transformations

