

# Exploring Abstract Algebra with Computer Software

## PREP Workshop 2004

### Section 4: Proof that $A_n$ is Simple

The computer exercises in this section are based on material written by Christine Stevens at Saint Louis University. We will use GAP to help us prove that  $A_6$  is a simple group. The command `AlternatingGroup(n)` creates the group  $A_n$ . For example

```
gap> a4:=AlternatingGroup(4);
Alt( [ 1 .. 4 ] )
```

creates the group  $A_4$  in GAP. The command `ConjugacyClasses(G)` lists all the conjugacy classes of a group  $G$ . For example:

```
gap> ConjugacyClasses(a4);
[ ()^G, (1,2)(3,4)^G, (1,2,3)^G, (1,2,4)^G ]
```

For  $a \in G$ , the notation  $a^G$  above means the set of all conjugates of  $a$  in  $G$ . Thus we see that  $A_4$  has four conjugacy classes: the conjugates of the identity, the conjugates of  $(1,2)(3,4)$ , the conjugates of  $(1,2,3)$  and the conjugates of  $(1,2,4)$ . The command `ConjugacyClass(G,a)` creates the conjugacy class of  $G$  containing  $a$ :

```
gap> c:= ConjugacyClass(a4,(1,2,3));
(1,2,3)^G
gap> Elements(c);
[ (2,4,3), (1,2,3), (1,3,4), (1,4,2) ]
```

#### Section 4, Project

4.1 Suppose you have disjoint sets  $T, U, V, W, X, Y$  and  $Z$  with cardinalities 1, 40, 40, 45, 72, 72 and 90 respectively. Suppose  $H$  is a set that is formed by taking the union of  $T$  with one or more of the other sets. List all the possible cardinalities of  $H$ . Which of these answers divide 360?

4.2 Use GAP to find all the conjugacy classes of  $A_6$  and their cardinalities.

4.3 Let  $G$  be a group and  $H$  a normal subgroup of  $G$ . Let  $h \in H$ . Show the conjugacy class of  $h$  is a subset of  $H$ .

4.4 Use Exercises 4.1 - 4.3 to prove that  $A_6$  is simple.

4.5 Use similar techniques as above to show  $A_5$  is simple.

4.6 Show  $A_4$  is not simple.