

# Exploring Abstract Algebra with Computer Software

## PREP Workshop 2004

### Section 2: Direct Product of Groups

In this section you will need the file “orderFrequency”. Fetch this file from the workshop web site and place it in the folder “gap4r4”. You will need to read this file into GAP. (A print out of this file appears at the end of this section.) The command to form direct products is `DirectProduct`. For example:

```
gap> C4:= CyclicGroup(IsPermGroup,4);
Group([ (1,2,3,4) ])
gap> S3:=SymmetricGroup(3);
Sym([ 1 .. 3 ])
gap> D:= DirectProduct(S3,C4);
Group([ (1,2,3), (1,2), (4,5,6,7) ])
gap> Size(D);
24
gap> Read("orderFrequency");
gap> orderFrequency(D);
[[ 1, 1 ], [ 2, 7 ], [ 3, 2 ], [ 4, 8 ], [ 6, 2 ], [ 12, 4 ]]
```

The first command above assigns the name  $C4$  to the cyclic group of order 4 (generated by  $(1,2,3,4)$ ). The third command forms the direct product  $S3 \oplus C4$ . The next two lines tell us that  $S3 \oplus C4$  has 24 elements. From the last output above we see that  $S3 \oplus C4$  has one element of order 1, seven elements of order 2, two elements of order 3, eight elements of order 4, two elements of order 6, and four elements of order 12.

#### Section 2, Project

2.1 **By hand** find the number of elements of each order in  $D_{10} \oplus \mathbf{Z}_2$ .

2.2 Check your answer to Exercise 2.1 using `orderFrequency`.

2.3. Use `orderFrequency` to find the number of elements of each order in  $D_5 \oplus \mathbf{Z}_4$ .

2.4. Are  $D_{10} \oplus \mathbf{Z}_2$  and  $D_5 \oplus \mathbf{Z}_4$  isomorphic? Why or why not?

2.5. **By hand** find the number of elements of each order in  $D_{20}$ .

2.6. Check your answer to Exercise 2.5 using `orderFrequency`. Is  $D_{20}$  isomorphic to either  $D_{10} \oplus \mathbf{Z}_2$  or  $D_5 \oplus \mathbf{Z}_4$ ?

2.7. How many nonisomorphic groups of order 40 can you find?

The command `AllSmallGroups(n)` creates a list of all groups of order  $n$ .

```
gap> Size(AllSmallGroups(40));
14
gap> List(AllSmallGroups(40), x -> orderFrequency(x));
[[ [ 1, 1 ], [ 2, 1 ], [ 4, 2 ], [ 5, 4 ], [ 8, 20 ], [ 10, 4 ],
  [ 20, 8 ] ],
 [ [ 1, 1 ], [ 2, 1 ], [ 4, 2 ], [ 5, 4 ], [ 8, 4 ], [ 10, 4 ],
  [ 20, 8 ], [ 40, 16 ] ],
 [ [ 1, 1 ], [ 2, 1 ], [ 4, 10 ], [ 5, 4 ], [ 8, 20 ], [ 10, 4 ] ],
 [ [ 1, 1 ], [ 2, 1 ], [ 4, 22 ], [ 5, 4 ], [ 10, 4 ], [ 20, 8 ] ],
 [ [ 1, 1 ], [ 2, 11 ], [ 4, 12 ], [ 5, 4 ], [ 10, 4 ], [ 20, 8 ] ],
 [ [ 1, 1 ], [ 2, 21 ], [ 4, 2 ], [ 5, 4 ], [ 10, 4 ], [ 20, 8 ] ],
 [ [ 1, 1 ], [ 2, 3 ], [ 4, 20 ], [ 5, 4 ], [ 10, 12 ] ],
 [ [ 1, 1 ], [ 2, 13 ], [ 4, 10 ], [ 5, 4 ], [ 10, 12 ] ],
 [ [ 1, 1 ], [ 2, 3 ], [ 4, 4 ], [ 5, 4 ], [ 10, 12 ], [ 20, 16 ] ],
 [ [ 1, 1 ], [ 2, 5 ], [ 4, 2 ], [ 5, 4 ], [ 10, 20 ], [ 20, 8 ] ],
 [ [ 1, 1 ], [ 2, 1 ], [ 4, 6 ], [ 5, 4 ], [ 10, 4 ], [ 20, 24 ] ],
 [ [ 1, 1 ], [ 2, 11 ], [ 4, 20 ], [ 5, 4 ], [ 10, 4 ] ],
 [ [ 1, 1 ], [ 2, 23 ], [ 5, 4 ], [ 10, 12 ] ],
 [ [ 1, 1 ], [ 2, 7 ], [ 5, 4 ], [ 10, 28 ] ] ]
```

The following is a copy of the file “`orderFrequency`”:

```
orderFrequency:= function(g)
local h,w;
w:= [ ];
w:= h -> Collected(List(Elements(h), Order));
return w(g);
end;
```