Abstract Algebra with GAP
Section 1 Appendix: Three Representations of the Group of Rotations of a Cube

In this series of exercises we consider the group of rotations of the cube. We start by labeling the vertices of the cube and record their movement under various rotations. For example, a rotation of $90°$ about the center of the front face corresponds to the permutation $(1,2,3,4)(5,6,7,8)$ of the vertices. Likewise, a rotation of $90°$ about the center of the left face gives rise to the permutation $(1,5,6,2)(3,4,8,7)$.

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gap> G:= SymmetricGroup(8);
Sym( [1 ..  8] )
gap> f:=(1,2,3,4)(5,6,7,8);
(1,2,3,4)(5,6,7,8)
gap> l:=(1,5,6,2)(3,4,8,7);
(1,5,6,2)(3,4,8,7)
gap> K:= Subgroup(G, [f,l]);
Group([(1,2,3,4)(5,6,7,8), (1,5,6,2)(3,4,8,7)]);
gap> Size(K);
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Find the effect of following the rotation f by the rotation l.

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gap> f*l;
(2,4,5)(3,8,6)
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Note that vertices 1 and 7 are fixed by this rotation of order 3. Hence the rotation has axis the long diagonal of the cube that runs through vertices 1 and 7, and is a rotation of $120°$

Section 1 Appendix, Project 1:

1.1 Classify the elements of K by their types of axes of rotation.

1.2 Compute the following products in GAP, and give a geometric description of the rotation (i.e., describe the axis of rotation and the magnitude of the rotation).
a) f^2

b) f^2l^2
c) $flfl$
d) f^2l
e) flf

Another way to represent the rotations of the cube is to consider the effect of each rotation on the faces. The rotation f of 90° about the center of the front face corresponds to the permutation $(1, 5, 6, 3)$, while the rotation l of 90° about the center of the left face corresponds to the permutation $(1, 4, 6, 2)$.

Section 1 Appendix, Project 2:

1.3 Set up the group of rotations of the cube as a subgroup of S_6. Again classify the elements of this subgroup by their types of axes of rotation.
1.4 Use this representation to compute the following products in GAP, and give a geometric description of the rotation (i.e., describe the axis of rotation and the magnitude of the rotation).
a) f^3
b) f^3l^3
c) $flflf$
d) f^3l
e) $lflf$

1.5 Express the rotation $(1, 3, 4)(2, 6, 5)$ as a product of f's and l's.

1.6 Express the rotation $(1, 6)(2, 5)(3, 4)$ as a product of f's and l's.

A third way to represent the rotations of the cube is to consider the effect of each rotation on the four (long) diagonals.

Section 1 Appendix, Project 3:

1.7 Set up the group of rotations of the cube as a subgroup of S_4. Again classify the elements of this subgroup by their types of axes of rotation.
1.8 Use this representation to compute the following products in GAP, and give a geometric description of the rotation (i.e., describe the axis of rotation and the magnitude of the rotation).

a) l^2
b) $l^2 f^2$
c) $lflfl$
d) $f^3 l^2$
e) $flfl^2$