Proof of Sylow's Theorem

[These transparencies illustrate the argument in Dummit and Foote's proof of Sylow's Theorem (Th. 18, p. 142) by applying it to the specific example of the alternating group A_5.]

```
gap> a5:=AlternatingGroup(5);
Alt( [ 1 .. 5 ] )
gap> ss3:=SylowSubgroup(a5,3);
Group([ (1,2,3) ])
gap> Size(ss3);
3
gap> Size(Normalizer(a5,ss3));
6
gap> ConjugateSubgroups(a5,ss3);
[ Group([ (1,2,3) ]),
Group([ (1,2,4) ]),
Group([ (1,2,5) ]),
Group([ (1,3,4) ]),
Group([ (1,3,5) ]),
Group([ (1,4,5) ]),
Group([ (2,3,4) ]),
Group([ (2,3,5) ]),
Group([ (2,4,5) ]),
```

```
Group([ (3,4,5) ]) ]

gap> ConjugateSubgroups(ss3,ss3);
[ Group([ (1,2,3) ]) ]
gap> th2:=Group([ (1,2,4) ]);
Group([ (1,2,4) ])
gap> ConjugateSubgroups(ss3,th2);
[ Group([ (1,2,4) ]),
Group([ (2,3,4) ]),
Group([ (1,4,3) ]) ]

gap> th3:=Group([ (1,2,5) ]);
Group([ (1,2,5) ])
gap> ConjugateSubgroups(ss3,th3);
[ Group([ (1,2,5) ]),
Group([ (2,3,5) ]),
Group([ (1,5,3) ]) ]

gap> th4:=Group([ (1,4,5) ]);
Group([ (1,4,5) ])
gap> ConjugateSubgroups(ss3,th4);
[ Group([ (1,4,5) ]),
```

```
Group([ (2,4,5) ]),
Group([ (3,4,5) ]) ]
```