Projects about Groups

[As the class was completing the portion of the course that dealt with groups, they were assigned projects, on which they could work singly or in pairs. The purpose was to review group theory by having them apply all the concepts that we had studied to a particular group. Students were free to use GAP, although they were cautioned against over-use, and there were some project on which GAP was of minimal assistance. The students were given about ten days in which to complete their projects.

The students used GAP in a variety of ways. One of the most profitable uses of GAP was carried out by the two students who worked on Group #7, who were able, with the assistance of GAP, to show that GL(2,3) is isomorphic to the semidirect product of the symmetric group S_3 acting on the group of quaternions.]

Your very own group

As you know, one of the requirements for this course is a project. All projects have the same format: you will be given a group G. and your task is to investigate its properties, using the concepts in group theory that we have studied. Your goal is to understand your group as thoroughly as possible and to explain its properties as clearly as possible.

Your explanation will take the form of a written report (3 to 5 pages) on your group. For a start, you should convince yourself that your given group really <u>is</u> a group. Then you should explore its properties. While there is no fixed set of questions that you are required to answer, here are some suggestions about issues that might be worth investigating:

> What is the order of your group?
>
> What are the orders of it elements?
>
> Is your group abelian? cyclic?
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> Is your group finitely generated?
>
> What is the center of your group?
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> What subgroups does your group have? Which ones are normal? Are any of them abelian, cyclic, or isomorphic to "familiar" groups?
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> Is your group simple?
>
> What is the derived group of your group?
>
> Is your group solvable?

Does your group have a composition series?

What is the automorphism group of your group?

Is your group isomorphic to any "familiar" group?

Judicious use of GAP is permitted, but you should avoid basing your answers on GAP and GAP alone. For example, it is acceptable to use GAP to find the derived subgroup of G, but it is probably not acceptable to use the "IsSolvable(G)" command as your only justification for asserting that G is or is not solvable.

You are permitted to discuss your group with your classmates, but you are expected to do your own work and to write your own report. (If you are working as a team, only one report is necessary. It should include a statement, signed by both team members, indicating what contributions each individual made to the project.)

Group #1: $G = \mathbf{Z}_4 \times \mathbf{Z}_6$, with the group operation defined as follows: $(a,b)(c,d) = (a+c, b+d)$.

Group #2: $G = \mathbf{Z}_4 \times \mathbf{Z}_3$, with the group operation defined as follows:
$(a,b)(c,d) = (a+c, b+(-1)^a d)$.

Group #3: $G = \langle x,y : x^6=1, x^3y^{-2}=1, y^{-1}xyx=1 \rangle$

Group #4: $G = \langle x,y : x^3=1, y^3=1, (xy)^2=1 \rangle$

Group #5: $G = \langle x,y : x^4=1, x^2y^{-2}=1, y^{-1}xyx=1 \rangle$

Group #6: G is the set of all matrices of the form

$$\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$$

where $a, b, c \in \mathbf{Z}_5$.

Group #7: G is the set of all matrices of the form

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

where $a, b, c, d \in \mathbf{Z}_3$, and ad-bc does not equal 0.

Group #8: $G = \{x \in \mathbf{Z}_{60} : x$ has a multiplicative inverse in $\mathbf{Z}_{60}\}$, with the operation of multiplication.

Group #9: G is a non-abelian group of order 147 whose Sylow 7-subgroup is cyclic.

Group #10: $G = \{a/2^n : a, n \in \mathbf{Z}, n \geq 0\}$, with the operation of addition.

Group #11: $G = R^2$, with the group operation defined as follows: $(a,b)(c,d) = (a+e^bc, b+d)$.