Homework on group actions

[These problems were assigned about ten days after the session in the computer classroom in which students were introduced to GAP. They are straightforward computations that review important definitions, and they can easily be done by hand. The assignment, however, deliberately invites students to use GAP, if they wish. After doing the problems, one student observed in class that "it seems as though you don't really need GAP for these problems." This was, of course, the point -- that a computational algebra system isn't always necessary or useful, and that mindless reliance upon such a system is not helpful.

The notation in this assignment regards the dihedral group of order 8, denoted D_8, as generated by an element r of order 4 and an element s of order 2, with the relation $rs=sr^{-1}$.]

Note: Some of these questions will require you to perform computations in D_8, but some can be done by using the definitions, without actually doing any (or many) computations. If you want to use GAP, go right ahead.

1. Let D_8 act on itself by left multiplication.

    a. Find the kernel of this action.

    b. Find the stabilizer of 1.

    c. Find the stabilizer of r.

    d. Find the stabilizer of s.

    e. Find the orbit of 1.

    f. Find the orbit of r.

    g. Find the orbit of s.

    h. Is this action faithful?

    i. Is this action transitive?

2. Repeat #1 if D_8 acts on itself by conjugation.

3. If a group G acts on itself by conjugation, then the kernel of the action is a familiar subgroup of G, namely _____.

4. If a group G acts on itself by conjugation and a is an element of G, then the stabilizer of G is a familiar subgroup of G, namely _____.