

# Transparency on commutator subgroups

[This transparency was used in class to illustrate the GAP command for finding the derived subgroup  $[G, G]$  of a group. It also introduces the notion of the derived series and the theorem (Thm. 9, p. 197 of Dummit and Foote) that a group is solvable if and only if its  $n^{\text{th}}$  derived group is trivial, for some whole number  $n$ .]

```
gap> s5:=SymmetricGroup(5);
```

```
Sym( [ 1 .. 5 ] )
```

```
gap> g5:=DerivedSubgroup(s5);
```

```
Group([ (1,3,2), (2,4,3), (2,3)(4,5) ])
```

```
gap> Size(g5);
```

```
60
```

```
gap> DerivedSubgroup(g5);
```

```
Group([ (1,4)(2,3), (1,2,3), (2,5)(3,4) ])
```

```
gap> Size(DerivedSubgroup(g5));
```

```
60
```

```
gap> d10:=Group((1,2,3,4,5), (2,5)(3,4));
```

```
Group([ (1,2,3,4,5), (2,5)(3,4) ])
```

```
gap> Size(d10);
```

```
10
```

```
gap> g10:=DerivedSubgroup(d10);
```

```
Group([ (1,3,5,2,4) ])
```