Assignment on Centers, Centralizers, and Normalizers

[These problems are part of an assignment that was due a week after the class period that was spent in the computer classroom, during which students had been introduced to GAP [See handout on "Centralizers, Normalizers, and Centers with GAP."] The purpose of these problems is to encourage students to use GAP to formulate conjectures and find counterexamples; they had to support the conjectures with formal proofs. The assignment included other problems that did not require use of GAP.]

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1. Let G be a group, and let H be a subgroup of G. Use GAP help you investigate the following statements. ["Investigate" means deciding whether the statement is true or false. If the statement is false, give a counterexample; if the statement is true, give a proof of the statement.]

   a. If N is the normalizer of H in G, then N is a normal subgroup of G.

   b. If N is the normalizer of H in G, then H is a normal subgroup of N.

   c. If C is the centralizer of H in G, then C is a normal subgroup of G.

   d. If C is the centralizer of H in G, then H is a normal subgroup of C.

2. Let G be a group, H a subgroup of G, and h an element of H. Consider the following subgroups of G: H, Z(H), Z(G), $C_G(H)$, $N_G(H)$, <h>, and $C_G(h)$. Use GAP to help you decide which of these groups <u>must</u> be subgroups of one or more of the other groups. Draw a subgroup sublattice that shows the relationships between these groups, and prove any inclusion relationship that appears in your sublattice.